DOCUMENT RESUME

ED 075 210 24 SE 015 864

TITLE	Programmed Math Continuum, Level One, Algebra, Volume 9.
INSTITUTION	New York Inst. of Tech., Old Westbury.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-8-0157
PUB DATE	[73]
CONTRACT	OEC-0-8-080157-3691(010)
NOTE	170p.
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	*Algebra; *Computer Assisted Instruction; Curriculum; Individualized Instruction; *Instruction; Instructional Materials; Mathematics Education; Programed Instruction; Programed Materials; *Programed Texts; *Secondary School Mathematics

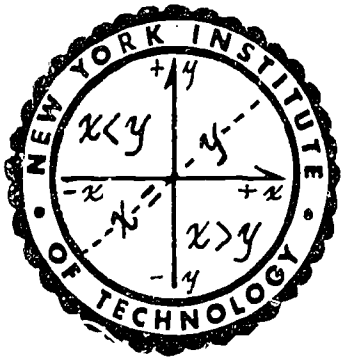
ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 9: factoring a trinomial square, sight multiplication of binomials, factoring the product of a binomial sum or difference, general factoring of a quadratic, and combining types of factoring. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

PROGRAMMED MATH CONTINUUM

level one

ALGEBRA



VOLUME

9

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME 9

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 9

TABLE OF CONTENTS

VOLUME 9

This volume covers the following material as shown in this excerpt from the Syllabus:

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Factoring a trinomial square	7- 6	11-7	8-7
2	Multiplying binomials at sight	7- 7	11-6	8-8
3	Factoring: prod. binomial sum-diff.	7- 8 7- 9	11-7	8-8
4	General factoring-quadratic trinom.	7-10	11-7	8-8
5	Combining types of factoring	7-11	11-8	8-8

READING ASSIGNMENT

VOLUME 9

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	251	253
2	253	255
3	255	259
4	259	260
5	261	263

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult: Algebra I

Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE.

These, too, should be entered in your NOTEBOOK.

HOMEWORK ASSIGNMENT

VOLUME NO. 9

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	252	4 , 5 , 6	09114
2	252	3 , 7 , 8	09116
3	253	19 , 23 , 24	09115
4	253	29 , 30 , 38 , 39	09117 , 09118
5	254	2 , 7 , 9	09216
6	254	3 , 4	09217
7	254	5 , 6 , 10	09218
8	254	12 , 13 , 18	09211 - 09218
9	256	2 , 3 , 4	09314
10	256 - 257	5 , 6 , 8 , 11	09316
11	258	3 , 4 , 8 , 13	09323
12	260	21 , 22 , 23	09320
13	260	2 , 5 , 6	09412 - 09413
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15	260	16 , 18	09412
16	260	20	09414
17	262	1 , 2 , 9	09512
18	262	23 , 25 , 26	09513 - 09514
19	262	38	09516
20	262	55	09516

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

VOLUME 9 SEGMENT 1 begins here:

Obtain a PUNCH CARD form your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	4 1	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 9	(Volume Number)
	66	and	68	0 1	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 251 - 253

SUPPLEMENTARY NOTES:

One of the most useful skills that you will need in your future work in algebra is the factoring of trinomials. It will aid you in reducing fractions and in simplifying complicated problems.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Recognize which of the following polynomials is a trinomial and select the letter next to the correct answer.

(A) $x^3 - 3x^2 + 3x + 3$

(B) $1 - 9y + 9y^2$

(C) $3xyz$

(D) None of these.

$\frac{2}{1}$

Very good. You made the correct choice.

Let's do this question together. The square root of $9x^2$ is $3x$, the square root of 49 is 7; and the middle term is twice the product of these 2 square roots. That is,

$$2 (3x) (7) = 42x$$

Please go on to question 5 below.

$\frac{2}{2}$

Question 5

The following trinomials are trinomial squares:

I. $n^2 + 18n + 81$

II. $144y^2 + 24y + 1$

Perform the necessary calculation and find the letter which expresses both trinomials as squares of binomials.

(A) I. $(n + 9)^2$, II. $(14y + 1)^2$

(B) I. $(n + 18)^2$, II. $(12y + 1)^2$

(C) I. $(n + 9)^2$, II. $(12y + 1)^2$

(D) I. $(n + 18)^2$, II. $(14y + 1)^2$

$\frac{3}{1}$

A trinomial indicates by its name the number of terms in the polynomial. In this case, there are three terms containing variables and a fourth term, the 3 . This makes the expression a quadrinomial.

Please return to page $\frac{18}{1}$ and try this question again.

$\frac{3}{2}$

We don't agree. Let us take a look at the trinomial

$$x^2 + 12x + 36$$

First ask yourself, "Is it a trinomial square?" The first term is squared,

$$x^2 = (x)^2$$

and so is the last term, $36 = 6^2$

Thus, the first and last terms are each perfect squares.

The middle term is $12x = 2(x)(6)$

This is, the middle term is twice the product of the square roots of the first and last terms.

Hence, $x^2 + 12x + 36$

is a trinomial square.

The question now is, "What is it the square of?"

Please return to page $\frac{20}{1}$ and try the problem again.

$\frac{4}{1}$

You did not [illegible]ht choice.

If you were working in a bank and you were counting a stack of 100 dollar bills, would you count the money more than once to make sure that you have the correct amount? Of course you would. In other words, in important situations, you check to make sure. Well, algebra is important too! Why don't you check your answer by multiplying (8p + 1) and (2p + 1) to see if you get

$$16p^2 + 8p + 1 \text{ ?!}$$

Please return to page $\frac{8}{1}$ and try this question again.

$\frac{4}{2}$

This is the correct answer. Be sure that you check your answer each time before you make your decision.

Please go on to question 9 below.

Question 9

Apply the proper principle and factor the trinomial,

$$64x^2 + 48xy + 9y^2$$

Select the letter which labels the correct answer.

(A) $(4x + 3y)^2$

(B) $(8xy + 3)^2$

(C) $(8x + 3y)^2$

(D) $(16x + 3y) (4x + 3y)$

A trinomial is an expression consisting of 3 terms. The coefficients of the terms, or the magnitude of the exponents, in no way determines whether an expression is a trinomial or not.

Please return to page 1/2 and choose another letter.

A necessary condition that a trinomial is a trinomial square is that the first term and the third term are each perfect squares. Now, is the number 20 a perfect square ? You know the answer to this question.

Please return to page 16/2 and try the question again.

$\frac{6}{1}$

You did not make the right choice. We can help you to do so by asking just one little question: What is 14 x 14 equal to?

Please return to page $\frac{2}{2}$ and try the question again.

$\frac{6}{2}$

Your method is all right, but your arithmetic is not. The square of 17 is not 169 . You may find it convenient to remember the key squares

$$10^2 = 100$$
$$15^2 = 225$$
$$12^2 = 144$$
$$20^2 = 400$$

Thus, 169 must be somewhere between the square of 12 and the square of 15 .

Return to page $\frac{20}{2}$ and try the question again.

We are sure that you made your choice hastily. Why should a trinomial consisting of the sum of cubes be called a trinomial square?

Return to page $\frac{18}{1}$ and try the question again.

In order for the expression,

$$9x^2 + \_\_\_\_\_ + 49$$

to be a trinomial square, it must equal

$$(3x + 7)^2$$

We are sure that you can square this binomial and discover what the middle term should be.

Return to page $\frac{19}{2}$ and choose another letter.

$\frac{8}{1}$

This choice is correct.

Now proceed to question 7 below.

Question 7

Apply the proper principle to write

$$16p^2 + 8p + 1$$

in factored form.

Select the letter which labels the correct answer.

(A) $(8p + 1) (2p + 1)$

(B) $(8p + 1) (8p + 1)$

(C) $(4p + 1) (4p + 1)$

(D) $(4p + 1) (4p - 1)$

$\frac{8}{1}$

We don't agree.

If a reward of $100 were offered for the correct answer, you would have checked your answer very carefully. You would have multiplied the binomials in each of the 4 choices to see if their product equalled the given trinomial,

$$64x^2 + 48xy + 9y^2$$

Please make believe that there is such a reward and check your choice.

Please return to page $\frac{4}{2}$ and try this question again.

We don't agree.

The expression you chose is a single term. A trinomial is an expression consisting of exactly 3 terms. For example,

$$15a^2bc$$

is a monomial, as it is an expression consisting of 1 term.

$$4xyz + 3x^2y$$

is a binomial, an expression of 2 terms.

$$5x - 3y + 7z$$

is a trinomial, an expression of 3 terms.

Please return to page 1/2 and choose another letter.

You chose the number 16. How do you know whether you made the correct choice? Let us suppose that you <u>did</u> make the correct choice.

Then, $(4r + 4)^2$

should equal $16r^2 + 40r + 16$

But, $(4r + 4)^2 =$

$16r^2 + \underline{32r} + 16$

Please reconsider, and before you make your decision; check your answer.

Please return to page 16/2 and try this question again.

$\frac{10}{1}$

Please multiply (n + 18) (n + 18) and tell us what you get.

You didn't get

$$n^2 + 18n + 81$$

did you?

Return to page $\frac{2}{2}$ and try this question again.

$\frac{10}{2}$

If you multiply the 2 binomials of your choice, you will find that the final term is $4n^2$ and not 4 . Please reconsider your choice and check your answer by first multiplying the 2 binomials.

Please return to page $\frac{20}{2}$ and try this question again.

We don't agree.

If you multiply the 2 binomials that you chose, you will get

$$(n^2 - 2n + 1)$$

Remember, n^4 is a square, the square of n^2. Please reconsider this problem.

Please return to page 21/1 and try this question again.

We are sorry that we have to disagree with you. The given trinomial is a trinomial square.

Please return to page 22/2 and try this question again.

$\frac{12}{1}$

You overlooked something. Reme- that a .inomial are is the result of squar.ng a binomial. When we square a binomial, the middle term is twice the produc= of the first and third terms.

Please return to page $\frac{18}{1}$ and try the question again.

$\frac{12}{2}$

You did not make the correct choice. Suppose that we solve a similar problem together. Express the trinomial,

$$25m^2 - 20mn + 4n^2$$

as a product of 2 binomials.

We first examine the given trinomial to see whether it is a trinomial square. Thus,

$$25m^2 = (5m)^2,$$

$$4n^2 = (2n)^2,$$

and $$20mn = 2(5m)(2n)$$

Hence, the given trinomial is a trinomial square. However, the middle term negative is the product of 2 like binomials, each connected with a minus sign. Thus,

$$25m^2 - 20mn + 4n^2 = (5m - 2n)(5m - 2n)$$

Please return to page $\frac{17}{2}$ and try this question again.

We don't agree.

You are confusing the square of a number with twice a number. Please reconsider your choice.

Please return to page 8/1 and try this question again.

One of the best ways of finding out whether you made the correct choice is to multiply the 2 binomials and see if their product equals the given trinomial,

$$y^6 - 8y^3 + 16$$

Watch those exponents!

Now, checking an answer is really a good idea. It makes you independent. You don't have to wait for the teacher to find out whether you are right or wrong; you can find out for yourself.

Please return to page 35/1 and try this question again.

$\frac{14}{1}$

Examine all the choices again. One of them does represent the correct answer.

Return to page $\frac{1}{2}$ and try this question again.

$\frac{14}{2}$

We don't agree.

For a trinomial to be a square trinomial, it is necessary that the last term is a perfect square.

Return to page $\frac{16}{2}$ and try this question again.

We don't agree.

You must have picked this letter without trying to do the question. You know that 18 x 18 does not equal 81.

You know 14 x 14 does not equal 144. Always check your answers in problems of this type by multiplying the factors to see if they do equal the trinomial given.

Please return to page 2/2 and try this question again.

If you will multiply the 2 binomials of your choice, you will find that the final term is 4mn not 4 .

Please reconsider your choice and before you decide on an answer, check by multiplying the 2 binomials.

Please return to page 20/2 and try this question again.

16
1

Your choice is correct. Let us discuss a method for testing a trinomial to see whether it is a square; that is, whether it is a trinomial obtained by squaring a binomial. We recognize a trinomial square in the following way:

Let us consider a trinomial;

$$4x^2 + 12x + 9$$

You ask yourself the following questions:

Is the first term a perfect square? Yes. $4x^2 = (2x)^2$

Is the last term a perfect square: Yes. $9 = (3)^2$

Is the middle term <u>twice</u> the product of the square roots of the first and third term? In other words, is

$$12x = 2(2x)(3)\ ?$$

Yes, it is. Therefore,

$$4x^2 + 12x + 9$$

is a trinomial square.

Please proceed to question 3 below.

16
1

<u>Question 3</u>

Perform the necessary calculation to fill in the missing term which will make the expression

$$16r^2 + 40r + \_\_\_\_\_$$

a trinomial square.

Select the letter which labels the correct statement.

(A) 25 (C) 16

(B) 20 (D) 5

Very good. You made the correct choice. Let us do this question together. The trinomial,

$$64x^2 + 48xy + 9y^2$$

is a trinomial square, since

$$64x^2 = (8x)^2$$

and $9y^2 = (3y)^2$

and $48xy = 2(8x)(3y)$

Hence, the factors are $(8x + 3y)(8x + 3y) = (8x + 3y)^2$

Please go on to question 10 below.

Question 10

Apply the proper principle and write the trinomial square,

$$16x^2 - 24xy + 9y^2$$

as a product of 2 binomials. Select the letter which labels the correct statement.

(A) $(4x + 3y)(4x + 3y)$

(B) $(16x - 9y)(x - y)$

(C) $(4x - 3y)(4x - 3y)$

(D) $(4x - 3y)(4x + 3y)$

18
1

This is the correct answer.

Please go on to question 2 below.

Question 2

Recognize which of the following polynomials is a trinomial square and select the letter next to the correct answer.

(A) $x^2 + y^2 + z^2$

(B) $a^3 + b^3 + c^3$

(C) $m^2 + mn + n^2$

(D) $x^2 + 10x + 25$

18
2

The product of 2 binomials that differ only in the sign between their terms is a binomial, because the "middle term" drops out.

For example,

$$(3x + 1)(3x - 1) =$$
$$9x^2 + 3x - 3x - 1 =$$
$$9x^2 - 1$$

Remember that to factor a binomial square factors into the product of 2 <u>like</u> binomials.

Return to page 8/1 and try the question again.

The square root of the first term is $4r$. Since the middle term must be twice the product of the square roots of the first and last terms, we have;

	2	x	$\sqrt{\text{first term}}$	x	$\sqrt{\text{last term}}$	= middle term
Substituting:	2		(4r)		$\sqrt{\text{last term}}$	= 40r
or	8r				$\sqrt{\text{last term}}$	= 40r
dividing by 8r					$\sqrt{\text{last term}}$	= 5
Squaring					last term	= 25

This choice is correct.

Now proceed to question 4 below.

Question 4

Perform the necessary calculation to fill in the missing term which will make the expression

$$9x^2 + \_\_\_\_\_ + 49$$

a trinomial square. Select the letter which labels the correct choice.

(A) $36x$

(B) $45x$

(C) $42x$

(D) $49x$

$\frac{20}{1}$

This is the correct answer.

Please go on to question 6 below.

Question 6

Apply the proper principle and write

$$x^2 + 12x + 36$$

in factored form. Select the letter which labels the correct answer.

(A) $(x + 4) (x + 9)$

(B) $(x + 6) (x + 6)$

(C) $x(x + 12) + 36$

(D) $(x + 18)^2$

$\frac{20}{2}$

This is the correct answer.

Please go on to question 8 below.

Question 8

Apply the proper principle to write

$$169m^2n^2 + 52mn + 4$$

in factored form.

Select the letter which labels the correct statement.

(A) $(17mn + 2) (17mn + 2)$

(B) $(13mn + 2) (13mn + 2)$

(C) $(13m + 2n) (13m + 2n)$

(D) $(13m + 2n) ($ [illegible] $+ 2n$

This is the correct answer

Please turn to the next question.

Question 11

Apply the proper principle and write

$$n^4 - 2n^2 + 1$$

in factored form.

Select the letter which labels the correct statement.

(A) $(n - 1)(n - 1)$

(B) $(n^2 - 1)(n^2 - 1)$

(C) $n^2(n^2 - 1) + 1$

(D) $(n^3 - 1)(n - 1)$

You did not make the correct choice. You should say to yourself:

"If $k = 1$

is the right answer,

then $(n - 1)^2$

should equal $n^2 - 16 + 1$

But, this is not the case, therefore, I must be wrong."

Please return to page 21/2 and try this question again.

22
1

Very good. You made a correct choice. The fact that the first term is y^6 did not fool you. We have, in fact, a trinomial square since

$$y^6 = (y^3)^2$$

$$16 = (4)^2$$

and $$8y^3 = 2(y^3)(4)$$

Thus, since the middle term is negative, the factors are

$$(y^3 - 4)(y^3 - 4)$$

Please go on to question 13 below

22
2

Question 13

Apply the proper principle and write the expression

$$x^2 + \frac{bx}{a} + \frac{b^2}{4a^2}$$

as a product of 2 binomials.

Select the letter which labels the correct statement.

(A) $(x + \frac{b}{a})(x + \frac{b}{a})$

(B) It is not a trinomial square and cannot be factored.

(C) $(x + \frac{b}{2})(x + \frac{b}{2})$

(D) $(x + \frac{b}{2a})(x + \frac{b}{2a})$

We grant you that this is a more difficult problem. But, it is always more fun to be able to do these harder problems. Let us give you a start. Think of the binomial

$$(x - 2y)$$

as a single term. You may even go one step further and let

$$(x - 2y)$$

equal to a single letter. Thus, if you wish, let

$$z = x - 2y$$

Now, the expression that you have to factor becomes,

$$z^2 - 6z - 2y$$

We are sure that you can continue from here without our help.

Please return to page $\frac{39}{2}$ and try this question again.

It appears that you are a little careless when it comes to signs. Recheck your work, as you should every time.

Return to page $\frac{37}{1}$ and choose the correct answer.

24
1

You did not make the right choice.

The first term of the product is the product of the first term of each of the binomials. That is, the first term equals

(5mx) ( )

To find this product, work your way from left to right, as,

(5) 3 equals?

(m) x^2 equals?

(x) x) equals?

Please return to page 42 and try question 3 again.

24
2

You know better than that!

The first term of the product of 2 binomials is the product of the first terms. When we multiply numbers having the same base, we keep the base and add the exponents. This you did, but when we multiply two arithmetic numbers, we do not add them.

Thus, 9 x 7 does not equal 16

Please return to page 38 and try this question again.

You did not make the correct choice. If we examine the given trinomial,

$$s^2 + ks + 9$$

we note that the first and last terms are squares. What must be the value of the middle term? We can write the following equation:

$$ks = 2 (\ ) (\ )$$

Fill in the numbers that belong inside the empty parentheses, and you will have the answer.

Please return to page 25/1 and try this question again.

We don't agree. Since you will come across this type of question in your future work in algebra, let's see if we can settle this type of problem once and for all. We are trying to find the value of k that will make the trinomial a trinomial square. We call this operation "completing the square." Here is a simple rule that does the trick.

> Take 1/2 the coefficient of the middle term and square the result you get.

Let us apply this rule to another trinomial, say

$$y^2 + 12y + k$$

One half the coefficient of the middle term is 1/2 (12) = 6. The square of 6 is 36. Thus,

$$y^2 + 12y + 36$$

is a trinomial square.

Please return to page 29/2 and try this question again.

To write an expression in factored form means to write the expression as a product of simpler expressions. Now, the choice you made is equivalent to the given trinomial,

$$n^2 - 2n^2 + 1$$

but this choice is a sum and not a product. To get started examine the given trinomial; see whether it is a trinomial square, and if it is, it can be written as the product of 2 like binomials.

Please return to page 21 and try this question again.

You have overlooked something. If you multiply the 2 binomials that you have chosen, you should find what you left out.

Return to page 22 and try this question again.

Not quite. This is a more difficult problem, granted, but the principle is still the same.

It is similar to

$$z^2 - 6z + 3$$

Now which would you say is the correct factored form,

$$(z - 3)^2$$

or $$(z + 3)^2 \ ?$$

Please return to page 39/2 and try the question again.

Yes. We agree with you that the last term of the product of two binomials is the product of the last terms of the binomials.

However, $(-3b)(7b)$

does not equal $-21b$

You must have answered hastily, because we are sure that you remember the rule:

When we multiply two numbers with the same base, we add the exponents. Furthermore, an algebraic term without a visible exponent means that the term is of power one; that is, x means x^1.

Please return to page 48/1 and try the question again.

$\frac{28}{1}$

We don't agree.

The square of $2a$ means

$$(2a)\ (2a)$$

Now, $(a)\ (a)$

does not equal a.

We cannot believe that you have forgotten the rule for multiplying numbers which have the same base.

$$x^a \cdot x^b = x^{a+b}$$

$$x \cdot x = x^2$$

Return to page $\frac{37}{1}$ and try question 2 again.

$\frac{28}{2}$

You can't let your attention wander when doing algebra.

It's not likely that you have forgotten how to multiply two numbers having the same base. Remember, keep the base and add the exponents.

Please return to page $\frac{38}{2}$ and try this question again.

Very good. You made the correct choice.

If we examine the given trinomial, we note that the first term is

$$s^2 = (s)^2$$

The last term is $9 = (3)^2$

Now, the middle term must equal twice the product of the square roots of the first and last terms of the given trinomial. Thus,

$$ks = 2(s)(3)$$

$$ks = 6s \quad \text{(divide by s)}$$

$$k = 6$$

Please proceed to question 15 below.

Question 15

Apply the proper principle and find the value of k which makes the trinomial

$$n^2 - 16n + k$$

a trinomial square.

Select the letter which labels the correct statement,

(A) $k = 1$

(B) $k = 4$

(C) $k = 8$

(D) $k = 64$

The product of 2 binomials can be calculated by the method used in multiplying any two polynomials. We have learned how to do this only a short while ago. However, let's refresh your memory with an example.

Multiply $(5x + 3)(3x - 8)$. Write

$$\begin{array}{r} 5x + 3 \\ \underline{3x - 8} \\ 15x^2 + 9x \\ \underline{\quad - 40x - 24} \\ 15x^2 - 31x - 24 \end{array}$$

Multiply 3x and 5x to get $15x^2$.

Multiply 3x and 3 to get 9x .

Multiply -8 and 5x to get -40x .

Multiply -8 and 3 to get -24 .

By adding them, we obtain, $15x^2 - 31x - 24$.

Please return to page 34/2 and try this question again.

We don't agree.

The last term of the product of two binomials is the <u>product</u> of the last terms of the binomials. For example, the last term of the product $(7z - 5m)(3z + 9m)$ is $(-5m)(9m)$ which equals: $-45m^2$.

Please return to page 48/1 and try this question again.

Ask yourself the following question:

Is $n^4 - 2n^2 + 1$

a trinomial square?

If the answer to this question is yes, then the factors of this trinomial are 2 binomials that are exactly alike.

Please return to page 21/1 and try this question again.

It looks like you had the right idea, but reconsider the constant term

$$\left(\frac{b^2}{4a^2} \right)$$

What is the square root of that expression?

Please return to page 22/2 and try the question again.

32/1

The product of two binomials consisting of the sum and difference of the same two terms is a binomial, not a trinomial.

Thus, for example,

$(3x + 2y) (3x - 2y)$ is a binomial.

$(5x + 7) (5x - 7)$ is a binomial.

Return to page 37/2 and try this question again.

32/2

We don't agree. If instead of inner you think of closest, and instead of outer, you think of farthest, it may help you.

Let us illustrate what we mean by a similar example. What is the inner and outer product of

$(11y - 3) (5y + 2)$?

The inner product is the product of the terms that are closest to each other, namely, $(-3) (5y) = - 15y$.

The outer product is the product of the terms that are the farthest apart, namely, $(11y) (2) = 22y$

Please return to page 45/2 and try this question again.

Congratulations! You did a pretty difficult problem correctly.

We are going to do this problem together with you. To simplify matters, let

$$z = x - 2y$$

and the given expression can be re-written as

$$z^2 - 6z + 9$$

This trinomial is clearly a trinomial square and in factored form it equals

$$(z - 3)(z - 3)$$

Now, replacing the value of z we get

$$(x - 2y - 3)(x - 2y - 3)$$

This is the end of the segment. Before going on to the next segment, make sure that you have the following items in your notebook:

(1) A complete explanation of how to recognize a trinomial square.

(2) The identities,

$$a^2 + 2ab + b^2 =$$

$$(a + b)^2$$

and $$a^2 - 2ab + b^2 =$$

$$(a - b)^2$$

You should now do problems 1 through 4 of the HOMEWORK ASSIGNMENT.

Turn in your PUNCH CARD and the answer matrix.

34
1

VOLUME 9 SEGMENT 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	4 2	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 9	(Volume Number)
	66	and	68	0 2	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 253 - 254

SUPPLEMENTARY NOTES

When a manufacturer has to produce one unit of an item, the time it takes to produce this item is not nearly as important as when the manufacturer has to produce 1000 units of this item. In the latter event, it becomes extremely important to create devices which will produce the parts that make up the item, very rapidly. Such devices are usually called " jigs ".

The operation of finding the product of two binomials belongs to this class. It is an operation that we have to do very frequently in our work in algebra. It is, therefore, very important for us to follow the procedure used by the manufacturer and create a " jig " , (in mathematics it is called an algorithim) a device that will enable us to find the product of two binomials rapidly and efficiently. This, in essence, is the purpose of this segment. We will begin by having you do problems in the regular manner and then introduce " short cuts " or " algorithims " to speed up the process.

34
2

Question 1

Apply the proper principle and find the product of the binomials,

$$(3y + 2) (5y - 7)$$

Select the letter which labels the correct answer.

(A) $15y^2 - 14$ (C) $15y^2 - 11y - 14$

(B) $15y^2 + 31y - 14$ (D) $8y - 5$

This choice is the correct answer. Please go on to the next question.

Question 12

Apply the proper principle and write,

$$y^6 - 8y^3 + 16$$

in factored form.

Select the letter which labels the correct statement.

(A) $(y^3 - 4) (y^3 - 4)$

(B) $(y^3 - 8) (y^3 - 2)$

(C) $(y^3 + 4)^2$

(D) $(y^4 - 4) (y^4 - 4)$

Just a quick look at your choice should tell you that you have the wrong answer.

The last term of a trinomial square must be perfect square. The number 8 is not a perfect square.

Please return to page 29/2 and try question 15 again.

$\frac{36}{1}$

Good, this is the correct answer.

The fact that the coefficients of the given trinomial are letters instead of numbers didn't confuse you. Let's review the thinking you probably did.

Now, is the trinomial,

$$x^2 + \frac{bx}{a} + \frac{b^2}{4a^2}$$

a trinomial square?

The first term is $x^2 = (x)^2$

The last term is $\frac{b^2}{4a^2} = \left(\frac{b}{2a}\right)\left(\frac{b}{2a}\right) = \frac{b^2}{2a}$

So far, so good!

Now, the middle term is

$$\frac{b}{a}x = 2(x)\left(\frac{b}{2a}\right)$$

The 2's cancel. Thus, the given trinomial is a trinomial square; therefore, the factors are:

$$\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right)$$

Please go on to question 14 below.

$\frac{36}{2}$

Question 14

Apply the proper principle and find the value of k which makes the trinomial

$$s^2 + ks + 9$$

a trinomial square.

Select the letter which labels the correct statement.

(A) $k = 3$

(B) $k = 6$

(C) $k = 9$

(D) $k = 0$

This choice is correct.

Please go on to question 2 which follows.

Question 2

Apply the proper principle and find the product of the binomials

$$(2a - 3b) (2a + 3b)$$

Select the letter which labels the correct answer.

(A) $4a^2 - 9b^2$

(B) $4a^2 - 12ab - 9b^2$

(C) $4a - 9b$

(D) $4a^2 + 12ab - 9b^2$

No, it looks like you misunderstood the question. You were asked to find the third term of the product after simplifying the first result of multiplication. The simplification results in a trinomial. The last term of that trinomial is the one that is sought.

Please return to page 48/1 and answer the question again.

Very good. You made the correct choice. The first term of the product of two binomials is the product of the first terms: the

$$(5mx) \quad (3m^2x)$$

Now, $5 \times 3 = 15$

$m (m^2) = m^3$

$x (x) = x^2$

therefore, $(5mx) \quad (3m^2x) = 15m^3x^2$

Hence $15m^3x^2$

is the correct answer.

Please go on to question 4 below.

Question 4

Apply the proper principle and select the letter which best completes the sentence: the first term of the product of the binomials,

$$(9a^2y - 2b) \quad (7a^3y^2 - 5b) \text{ is}$$

(A) $16a^5y^3$

(B) $63a^6y^2$

(C) $63a^5y^3$

(D) $72a^5y^3$

Your choice is correct.

To "complete the square," that is, to make the binomial

$$n^2 - 16n$$

into a square trinomial by adding a constant term, we must follow this procedure:

Take $\frac{1}{2}$ of the coefficient of n : $\frac{1}{2}$ (-16), (which is -8) square it (which is 64) and add it to the binomial

$$n^2 - 16n + 64$$

Now proceed to question 16 below.

Question 16

Relate to the proper principle and write the expression,

$$(x - 2y)^2 - 6(x - 2y) + 9$$

in factored form.

Select the letter which labels the correct statement.

(A) $(x - 2y)$ $(x - 2y + 9)$

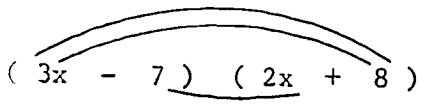
(B) $(x - 2y + 3)$ $(x - 2y + 3)$

(C) $(x - 2y - 3)$ $(x - 2y - 3)$

(D) None of these

40
1

Very good. You made the correct choice. Let us do this question together. The inner product of the two binomials



is the product of the two adjacent terms connected by the single loop; that is, $-7)\ (\ 2x\ ) = -14x$

The outer product is the product of the terms that are furthest apart, connected by the double loop; that is,

$$(\ 3x\ )\ (\ 8\ ) = 24x$$

Please proceed to question 7 below.

40
2

Question 7

Apply the proper principle to find the sum of the inner and outer product of

$$(\ 9x + 2\ )\ (\ 3x + 5\ )$$

Select the letter which labels the correct statement.

(A) 19x

(B) 51x

(C) 45x

(D) 51

You should use the F-O-I-L method to multiply two binomials. But it appears that you just multiplied the First and the Last and forgot about the outer and inner products.

Please return to page 51/2 and try this question again.

You have the right idea, but didn't you get mixed up on the sum of the outer and inner terms?

Return to page 56/2 and try the question again.

42/1

Correct! Let's review the proper procedure. If you examine the two binomials, you will notice that the terms in each of the binomials are exactly alike; namely,

2a and 3b

The only difference is that one binomial has a minus sign connecting the two terms, while the other has a plus sign. Now, such binomials can be multiplied at sight, using this rule:

"The product of the sum and difference of 2 terms equals the square of the first term minus the square of the second term."

This is an important rule which you cannot afford to forget. Please check whether you have it in your notebook. If not, write it down.

Memorize this proto-type $\boxed{(a+b)(a-b) = a^2 - b^2}$

Please proceed to question 3 below.

42/2

Question 3

Apply the proper principle and select the letter which best completes the sentence: The first term of the product of the binomials,

$(5mx - 7)(3m^2x + 4)$ is

(A) $15m^2x^2$

(B) $8m^2x^2$

(C) $15m^3x$

(D) $15m^3x^2$

You should review what inner product means. The inner product is the product of the two terms that are closest to each other. For example, in the product

$$(5y - 11) (3y + 8)$$

-11 and $3y$

are the terms that are closest.

Their product is called

the inner product

Similarly, $5y$ and 8

are the two terms that are the farthest apart. Their product is called the outer product. Keep this in mind.

Please return to page 57/2 and try this question again.

You made a hasty decision. Ask yourself again;

What does $(6p) (3p)$ equal?

What does $(-5q) (-2q)$ equal?

Keep in mind the laws of exponents as you decide upon the answer to these questions.

Please return to page 59/2 and try this question again.

$\frac{44}{1}$

Your algebra is fine, but your arithmetic is not. How much is 9 x 7 ?

Please return to page $\frac{38}{2}$ and try this question again.

$\frac{44}{2}$

Sorry, but this is not the right choice. Let us do a similar problem together.

Find the sum of the inner and outer product of the binomials

$$(5p - 3) (7p - 6)$$

The inner product is $(\quad - 3) (7p \quad) = 21p$

The outer product is $(5p \quad) (\quad - 6) = 30p$

The sum of the above two monomials is

$$- 51p$$

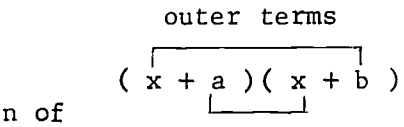
Note that the sign in front of the terms of the binomials, must be considered.

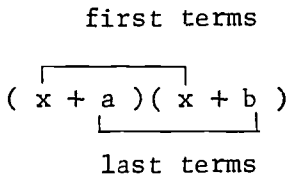
Please return to page $\frac{54}{1}$ and try this question again.

This is the correct answer.

Now let us introduce another method of multiplying two binomials:

First, notice the description of the positions of the terms:





Second, relate these to the product:

$$\begin{array}{r} x + b \\ \underline{x + a} \\ x^2 + bx \\ \underline{\quad + ax + ab} \\ x^2 + (ax + bx) + ab \end{array}$$

product of last term

sum of product of outer terms and product of inner terms

product of first terms

Question 6

Apply the proper principle and find (I) The inner product

(II) The outer product obtained by multiplying $(3x - 7)(2x + 8)$

Select the letter which labels the correct statement.

(A) I. $14x$, II. 24

(B) I. $-14x$, II. $24x$

(C) I. $6x^2$, II. -56

(D) None of these.

$\frac{46}{1}$

You did not make the right choice. You added when you should have multiplied. The inner product means the product of the two terms that are closest to each other. You took their sum. You made the same type mistake with the outer product.

Please return to page $\frac{40}{2}$ and try this question again.

$\frac{46}{2}$

Have you forgotten how to multiply two binomials?

There are four multiplications to be performed. It seems that you forgot two of them.

Please return to page $\frac{56}{2}$ and try this question again.

We don't agree. You have to include the sign of the terms.

The inner product is not merely

(9) (7x)

but (-9) (7x)

Please return to page 57/2 and try question 8 again.

Almost right. Please examine the last term of your choice, and see if you can find the mistake you made.

Return to page 51/2 and select another letter.

IX

48
1

This choice is correct. Please proceed to question 5 which follows.

Question 5

Apply the proper principle and select the letter which correctly completes the statement: The third term of the product of the binomials, after simplifying

$(y - 3b)$ $(y + 7b)$ equals,

(A) $-21b$

(B) $4b$

(C) $-21b^2$

(D) $-3by$

48
2

We don't agree.

Remember this important fact: the product of two binomials involves four separate multiplications. Didn't you forget the two middle products?

Return to page 59/2 and try this question again.

It is very important that you read the question carefully and answer exactly what is being asked. It is a shame to lose credit for a question when you really know how to answer it. Now, the question asked for the sum of the inner and outer product and your answer is the outer product only.

Please return to page 40/2 and try this question again.

We are disappointed. Don't you know that the area of a rectangle is the product of its length and width, and not the sum of the length and width?

Please return to page 61/2 and choose another letter.

50
1

You did not make the correct choice. One of the letters does have the correct answer next to it. Please reconsider.

Please return to page 57/2 and try this question again.

50
2

No, the answer is one of the other three choices.

Check your work carefully.

Return to page 56/2 and try the question again.

Very good. You made the correct choice. Let's do this question together.

We have to find the sum of the inner product and the outer product

$$(2y - 3) (7y - 5)$$

The inner product is $(\quad - 3) (7y \quad) = -21y$

The outer product is $(2y \quad) (\quad - 5) = -10y$

The sum is $-31y$

To multiply 2 binomials, we use the Distributive Law where each term of the first binomial multiplies each term of the second. In the above case.

$$(2y - 3) (7y - 5) =$$

$$2y (7y - 5) + (-3) (7y - 5)$$

or to list the steps:

(2y) (7y)	First Terms
(2y) (-5)	Outer Terms
(-3) (7y)	Inner terms
(-3) (-5)	Last terms

Students refer to this method as the "F-O-I-L" method from the initials.

Please go on to question 10 below.

Question 10

Apply the proper principle and multiply the binomials

$$(2m + 3) (5m + 4)$$

at sight.

Select the letter which has the same answer you got.

(A) $10m^2 + 12$

(B) $10m^2 + 23 + 12$

(C) $10m^2 + 23 + 7$

(D) My answer was different from any of the above.

You are confusing the area with perimeter. The area of a rectangle is the amount of surface it contains. It is equal to the product of its length and width. Think of it this way; the perimeter of a rectangle is the number of feet of fencing required to enclose it, while the area tells you how many things can fit inside the rectangle.

Please return to page 61/2 and try this question again.

It seems that you know the rule for multiplying binomials at sight, but you evidently have forgotten how to multiply numbers having the same base. When we multiply two numbers with the same base, we keep the base and add the exponents. Do you see your mistake?

Please return to page 64/1 and try this question again.

You lost something on the way. You forgot to include the letter. When you multiply

$$9x \text{ by } 5$$

the answer is not 45 .

Return to page 40/2 and try question 7 again.

You made an error in the middle term of your answer. The middle term of the product of two binomials is the <u>sum</u> of the inner and outer products.

Please return to page 59/2 and try this question again.

54
1

This is the correct answer. Please go on to the next question.

Question 9

Apply the proper principle and find the sum of the inner and outer product of

$$(2y - 3) (7y - 5)$$

Select the letter which labels the correct statement.

(A) -31

(B) -17y

(C) -11y

(D) -31y

54
2

We don't agree. Granted that you never had the problem of finding the product of 3 binomials at sight. However, you can forget about the third binomial for a minute, and consider the product of the first two binomials. After you find their product, you can then multiply this product by the third binomial. Look at it this way: a more complicated problem is usually only a combination of 2 or more simpler problems.

Please return to page 60/1 and try this question again.

It shouldn't be. The correct answer is listed. We checked our answer before writing it down. Did you check yours?

Return to page 5 and try question 10 again.

Sometimes a small experiment can be very helpful. May we suggest that you multiply the following binomials?

$$(x + 2)(x + 3)$$

$$(x + 2)(x - 3)$$

$$(x - 2)(x + 3)$$

$$(x - 2)(x - 3)$$

and examine each of the products for the sign between their terms. Have you arrived at a conclusion?

Return to page 66/2 and try this question again.

56
1

Very good! It takes courage and self confidence to say "my answer was different from any of the above." Let us do this problem together. We will use the FOIL method. The product of the two binomials

$$(6p - 5q)$$

and $$(3p - 2q)$$

is the product of the first two terms,

$$(6p) (3p) = 18 p^2$$

Then we get the sum of the outer and inner products; namely,

$$(-2p) (6q) - (-5q) (3p) =$$

$$-12pq - 15pq =$$

$$-27pq$$

Finally, we add the product of the last two terms,

$$(-5q) (-2q) = 10q^2$$

Hence, the answer is $18p^2 - 27pq + 10q^2$

Please go on to question 12 below.

56
2

Question 12

Apply the proper principle and multiply the binomials

$$(7t - 3) (4t + 5)$$

at sight. Select the letter which has the same answer that you obtained.

(A) $28t^2 - 47t - 15$

(B) $28t^2 - 15$

(C) $28t^2 + 23t - 15$

(D) My answer was different from any of the above.

This is the correct answer.

Let's review this problem

the outer product is (9x) (5) = 45x

the inner product is (2) (3x) = 6x

the sum of the two is = 51x

Now proceed to question 8 below.

Question 8

Apply the proper principle to find the sum of the inner and outer product of

(6x - 9) (7x + 5)

Select the letter which labels the correct statement.

(A) $42x^2 - 45$

(B) 33x

(C) -33x

(D) None of these.

$\frac{58}{1}$

Please remember that the product of two different binomials involves four multiplications.

Use the F-O-I-L method.

Return to page $\frac{61}{2}$ and choose another letter.

$\frac{58}{2}$

We don't agree.

The only hint that we are going to give you is to reconsider the middle term of your choice. We have confidence that you will find your error.

Please return to page $\frac{64}{1}$ and try this question again.

This is the correct answer.

$$(2m + 3) (5m + 4)$$

can be multiplied mentally by using the "F-O-I-L" method.

F: $(2m) (5m) = 10m^2$

O: $(2m) (+4) = +8m$

I: $(+3) (5m) = +15m$

} combining, we get $+23m$

L: $(+3) (+4) = +12$

Therefore, the answer is

$$10m^2 + 23m + 12$$

Please proceed to question 11 below.

Question 11

Apply the proper principle and multiply the binomials

$$(6p - 5q) (3p - 2q)$$

at sight. Select the letter which has the same answer that you obtained.

(A) $18p - 27pq + 10q$

(B) $18p^2 + 10q^2$

(C) $18p^2 - 3pq + 10q^2$

(D) My answer was different from any of the above.

60/1

This is the correct answer. Please go on to the next question.

Question 15

Without actually multiplying the binomials,

$$(x - 1) , (x + 1) , (x^2 + 3)$$

relate to the proper principles and find their continued product.

Select the letter which labels the correct statement.

(A) $x^4 - 3$

(B) $x^4 + 2x^2 - 3$

(C) $x^3 + 2x^2 - 3$

(D) $x^4 + 2x^2 + 3x - 3$

60/2

This choice is not correct. Why don't you substitute the values you've chosen for the blanks in the binomials and multiply them to check your answer.

You notice that the product of each pair of numbers must equal 12 .

Now,

$$7 \times 5 = 35$$

$$8 \times 4 = 32$$

$$6 \times 6 = 36$$

Since none of these products equals 12, every pair was wrong.

Please return to page 78/1 and make the correct choice.

Very good! You made the correct choice. In finding the product of two binomials at sight, we should say to ourselves, "If we need the product of the first two terms; that is,

$$(7t) (4t) = 28t^2$$

Then, we must add the sum of the inner and outer products,

$$(-3) (4t) + (5) (7t) =$$

$$-12t + 35t = 23t$$

Finally, we must add the product of the last two terms,

$$(-3) (5) = 15$$

Hence, the product is

$$28t^2 + 23t - 15 \text{ "}$$

Please proceed to question 13 which follows.

Question 13

The length and width of a rectangle are

$$(4x - 7)$$

and

$$(2x + 5)$$

respectively.

Apply the proper principle and express the area of this rectangle at sight. Select the letter which labels the correct statement.

(A) $6x - 2$

(B) $12x - 4$

(C) $8x^2 - 35$

(D) $8x^2 + 6x - 35$

62
1

Very good! You made the correct choice. If a trinomial with only plus signs between its terms is written as a product of two binomials, the binomials will have only plus signs between their terms.

Thus, for example,

$$x^2 + 11x + 30 = (x + 6)(x + 5)$$

Please go on to question 2 below.

62
2

Question 2

Perform the necessary calculation to find the number of different pairs of natural numbers whose product is 36 .

Select the letter which has the same answer that you obtained.

(A) 4 pairs

(B) 9 pairs

(C) 5 pairs

(D) 10 pairs

You selected a pair of numbers whose product is 28 . That is fine. But, there is an additional requirement. The pair of numbers must add up to the coefficient of x . Your selection does not meet this second requirement. Please do something about this.

Return to page 71/2 and answer the question again.

Let us consider all the possible factors of 24 .

They are,

(24 , 1) , (12 , 2) , (8 , 3) , (6 , 4)

Now, which of these factors will result in the smallest value of k ? A good idea would be to try each one and see.

Please return to page 73/2 and re-consider the problem.

64
1

This is the correct answer. Please go on to the next question.

Question 14

Apply the proper principle and write the product of the binomials,

$$(2x^2 - 3y^2) \text{ and } (4x^2 + 5y^2)$$

at sight.

Select the letter which labels the correct statement.

(A) $8x^2 - 2x^2y^2 - 15y^2$

(B) $8x^4 + 2x^2y^2 - 15y^4$

(C) $8x^4 - 2x^2y^2 - 15y^4$

(D) $8x^4 - 15y^4$

64
2

This choice is not correct.

If you recall, the product of the "last terms" should equal the constant term in the trinomial. Now since the pairs of numbers that you chose, actually become the "last terms" in the two binomials, shouldn't you multiply them as a test? At this point we are not concerned with the coefficient of the x term; we are considering only the constant term.

Return to page 78/1 and try the problem again.

Very nice! You made the correct choice. We'll enjoy doing this problem together with you. We start by examining the first two binomials, and we notice that they are exactly alike except for the sign between their terms. The product of such binomials is the square of the first term minus the square of the second term. Thus,

$$(x - 1)(x + 1) = x^2 - 1$$

Hence, it remains to find the product of

$$(x^2 - 1)(x^2 + 3)$$

The product of the two first terms is

$$(x^2)(x^2) = x^4$$

The sum of the inner and outer products is

$$(-1)(x^2) + (3)(x^2) = 2x^2$$

The product of the last terms is

$$(-1)(3) = -3$$

This is the end of the segment. Before going on to the next segment, you should have the following item in your notebook:

To multiply two binomials at sight, multiply the first terms of the binomials.

Multiply the first term of each binomial by the last term of the other.

Multiply the last terms of the binomials.

This can be expressed as a formula:

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

A quadratic term is a term of degree two.

A linear term is a term of degree one.

A constant term is a numerical term with no variable factor.

You should now do problems 5 through 8 of the HOMEWORK ASSIGNMENT.

Please turn in your PUNCH CARD.

66
1

VOLUME 9 SEGMENT 3 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	4 3	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 9	(Volume Number)
	66	and	68	0 3	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 255 - 258

SUPPLEMENTARY NOTES:

In this SEG you will learn how to do the reverse operation of what you learned in SEG 2 . In the previous SEG you learned how to multiply two binomials at sight to produce a trinomial; whereas in this SEG you will be given the trinomial and will be asked to find the two binomials that it equals. This operation is known as FACTORING THE TRINOMIAL.

In your NOTEBOOK you should have definitions, from the READING, for the following concepts: quadratic term, linear term, constant term, factor, prime trinomial, integral coefficient, and absolute value.

Now proceed to the first question, below. Good Luck!

66
2

Question 1

Apply the proper principle and select the letter which correctly completes the statement: If a trinomial with only plus signs between its terms, is written as a product of 2 binomials, then

(A) The binomials have minus signs between their terms.

(B) The binomials have plus signs between their terms.

(C) One binomial has a plus sign, and the other has a minus sign.

(D) The first binomial must have the plus sign, the second has the minus sign.

We don't agree.

Remember that the number 1 is a factor of every natural number.

Please reconsider your choice.

Return to page 62/2 and try this question again.

This is the correct answer. Please go on to the next question.

Question 8

Apply the proper principle and write the trinomial,

$$p^2 + 14p + 33$$

in factored form.

Select the letter which labels the correct answer.

(A) $(p + 13)(p + 3)$

(B) $(p + 11)(p + 3)$

(C) $(p - 11)(p - 3)$

(D) It can't be factored

$\frac{68}{1}$

Anybody can make a mistake, but when one makes the same mistake again and again, then there is something wrong. Now, give us your undivided attention.

Unless the binomials are in the form $(a + b)(a - b)$, where the product is $a^2 - b^2$, the product of two binomials is not a binomial but a trinomial.

The answer to this problem should contain three terms. Our purpose is to prevent you from making this same mistake again.

Please return to page $\frac{64}{1}$ and select another letter.

$\frac{68}{2}$

No, that answer is not correct.

Although it is true that there is an unlimited set of number pairs that have a product of 12,

such as $24 \times \frac{1}{2}$ $36 \times \frac{1}{3}$

and $1200 \times \frac{1}{100}$

the number of pairs of <u>integers</u> that have a product of 12 is definitely limited.

Return to page $\frac{78}{1}$ and try the problem again.

Yes,

$$9 + 2 = 11$$

but no, they are not factors of 28 , since 9 x 2 does not equal 28 .

Can you find a pair of factors that meet both requirements? Of course, you can.

Return to page 71/2 and try this question again.

Your choice shows that you overlooked an important clue.

Remember the following: If a trinomial has a plus sign before its last term, both binomial factors must have the same sign between their terms. When the middle term of the trinomial has a minus sign, both binomial factors have minus signs between their terms. To illustrate,

$x^2 - 5x + 6$ must be of the form

$(x - a) (x - 6)$

Now, return to page 74/2 and try this question again.

IX

70/1

How did you get 7 ? That is more than we got.

For 36, one pair would be 2 and 18 . We are counting the pairs not the different factors.

Please return to page 62/2 and try this question again.

70/2

Your best weapon in determining whether given factors of a trinomial are the correct ones, is your skill in finding the product of 2 binomials at sight. By this time, you should be an expert in this operation. Please examine the sum of the inner and outer products.

Please return to page 67/2 and try this question again.

Very good. You made the correct choice.

If	x^2	+	ax	+	12	
is to equal	(x	+	b)(x +		c)	
then	x^2	+	ax	+	12	
must equal	x^2	(bx	+	cx)	+	bc
or	x^2	+	(b + c)x		+	bc

Therefore, by comparison

	bc
must equal	12

Question 4

Apply the proper principle and select the letter which correctly completes the statement.

If $x^2 + kx + 28$

is written as the product of two binomials, the factors of

28

that must be used in order that

$k = 11$

are

(A) (14 , 2)

(B) (9 , 2)

(C) (28 , 1)

(D) (7 , 4)

$\frac{72}{1}$

If you can find two numbers whose product is 10 , and whose sum is 7 , the trinomial can be factored. Keep looking and you shall find such a pair of numbers.

Please return to page $\frac{84}{2}$ and choose another letter.

$\frac{72}{2}$

The best way to answer a question like this is to put the values of k into the trinomial

$$x^2 + 7x + k$$

and see if it can be factored. For example, if

$$k = 9$$

we get the trinomial $x^2 + 7x + 9$

The two possible pairs of factors of 9 are

$$(3, 3) \text{ and } (9, 1)$$

Using the first pair, we get

$$x^2 + 6x + 9$$

Using the second pair, we get

$$x^2 + 10x + 9$$

Thus, $k = 9$

is not a correct choice. Proceed in the same way with the other choices, and you will eventually come up with the correct values of k .

Please return to page $\frac{88}{2}$ and try this question again.

Very good. We are always pleasantly surprised when a student has the courage to choose "none of these". It shows that he has confidence in his work. By the way, do you know the correct factors of

$$6x^2 + 23x + 20 \text{ ?}$$

They are $(3x + 4)(2x + 5)$

In the next segment, we will learn how to factor such trinomials. Check the problem by using the F-O-I-L method.

$$(3x + 4)(2x + 5) = 3x(2x + 5) + 4(2x + 5)$$

F: (First): $(3x)(2x) = 6x^2$

O: (Outer): $(3x)(5) = 15x$

I: (Inner): $(4)(2x) = 8x$

L: (Last): $(4)(5) = 20$

Combining these we get:

$$6x^2 + 15x + 8x + 20$$

or $$6x^2 + 23x + 20$$

Please proceed to question 6 below.

Question 6

Apply the proper principle to find the smallest possible positive value of k , if

$$x^2 + kx + 24$$

is to be written as the product of 2 binomials. Select the letter which labels the correct statement.

(A) $k = 8$

(B) $k = 12$

(C) $k = 11$

(D) None of these.

$\frac{74}{1}$

You have too many factors. Perhaps you counted factors such as 3 , 12 and 12 , 3 as two different pairs. We are not counting the same numbers in reverse order as different pairs.

Please return to page $\frac{62}{2}$ and try this question again.

$\frac{74}{2}$

This is the correct answer.

Please proceed to question 9 which follows.

Question 9

Apply the proper principle and write the trinomial

$$y^2 - 15y + 56$$

in factored form. Select the letter which has the same answer that you obtained.

(A) (y + 9) (y - 6)

(B) (y - 9) (y + 6)

(C) (y - 14) (y - 4)

(D) (y - 7) (y - 8)

It is not enough that

$$28 \times 1 = 28$$

Please check your choice by multiplying the binomials

$$(x + 28) (x + 1)$$

and convince yourself that you did not make a correct choice.

Return to page 71/2 and choose another letter.

You should know better than that!

When a trinomial has only plus signs between its terms, then the binomial factors also have only plus signs between their terms. Please take time out and write this fact into your notebook.

Return to page 67/2 and try this question again.

76
1

You have made a wrong choice if you can find two integers whose product is 28 and whose sum is 11 . Try writing down all pairs of factors of 28 and see what happens!

Return to page 84 and try this question again.
2

76
2

Ask yourself this question, "If the sign between the terms of each binomial is negative, what will be the sign of the last term of the trinomial?" Now look at the trinomial given and reconsider the problem.

Please return to page 81 and answer the question again.
2

Very good. "None of these" is the right answer. If we consider all the possible factors of 24 ; namely,

(24 , 1) , (12 , 2) , (8 , 3) , (6 , 4)

and write all the binomials, we get,

(x + 24) (x + 1)

(x + 12) (x + 2)

(x + 8) (x + 3)

(x + 6) (x + 4)

Clearly, the last 2 binomials give us the correct answer.

Multiplying these binomials, we get

$$x^2 + 10x + 24$$

and $k = 10$

is the smallest value of k .

Please proceed to question 7 below.

Question 7

Apply the proper principle and select the value of k for which the trinomial

$$x^2 + kx + 18$$

cannot be written as a product of 2 binomial sums.

(A) $k = 11$

(B) $k = 19$

(C) $k = 9$

(D) $k = 10$

78
1

This is the correct answer.

Please [illegible] to question 3 below.

Question 3

Apply the proper principle and select the letter which correctly completes the statement.

If $x + ax + 12$

were written as $(x + \_\_\_)(x + \_\_\_)$

the pairs of integers that could appear above the dashes are:

(A) (7, 5) , (8, 4) , (6, 6)

(B) (1, 12) , (2, 6) , (3, 4)

(C) (4, 3) , (6, 1) , (5, 2)

(D) An infinite number.

78
2

Please check the sum of the inner and outer product of the two binomials that you chose.

Please return to page 74/2 and try this question again.

This is the correct answer.

Let us consider the theory and more. We know that

$$(x + a)(x + b)$$
$$= x^2 + ax + bx + ab$$
$$= x^2 + (a + b)x + ab$$

Note: The a and b that are factors of the constant term must also have a sum that is the coefficient of the line or term since 7 and 4 are the values given for a and b .

We note that $a \cdot b = 7 \cdot 4 = 28$

and $a + b = 7 + 4 = 11$

and these answers agree with the conditions given.

Please go on to question 5 below.

Question 5

Perform the necessary multiplications to determine the correct factors of

$$6x^2 + 23x + 20$$

Select the letter which labels the correct answer.

(A) $(6x + 5)(x + 4)$

(B) $(3x - 4)(2x - 5)$

(C) $(3x + 5)(2x + 4)$.

(D) None of these.

$\frac{80}{1}$

You say that it can't be factored, and we say that you give up too easily. Giving up too easily is not a good attitude in any situation, not only in algebra. Examine each of the choices carefully, and you will change your mind.

Please return to page $\frac{67}{2}$ and make the correct choice.

$\frac{80}{2}$

Do you know what a prime trinomial is? It is a trinomial that cannot be factored over the set of integers. If you try a little harder, you will find out that

$$x^2 + 6x + 9$$

can be factored.

Please return to page $\frac{84}{2}$ and try this question again.

Very good. You made the correct choice. If k equals 6 , we have

$$x^2 + 7x + 6 =$$

$$(x + 6) (x + 1)$$

If k = 10 , we have

$$x^2 + 7x + 10 =$$

$$(x + 5) (x + 2)$$

Please go on to question 12 below.

Question 12

Apply the proper principle and select the letter which completes this statement correctly. If the trinomial

$$z^2 + 2z - 15$$

is written as the product of two binomials, the signs between their terms are:

(A) Both negative

(B) Both positive

(C) One positive and one negative

(D) Any combination of signs is possible

82
1

We're sorry, but you did not make the correct choice. Consider the sum of the inner and outer products and you will discover your mistake.

Please return to page 95 and try this question again.
2

82
2

We caught you. You made a hasty choice and forgot all about the letter y . Clearly, the letter y must appear in the factors of a trinomial which has the letter y in it.

Please return to page 98 and try this question again.
1

You did not make the right choice.

If k = 11, the trinomial becomes

$$x^2 + 11x + 18$$

This trinomial can be written as

$$(x + 9)(x + 2)$$

Please return to page 77/2 and reconsider the question.

Be merely examining the choice you made, you should realize that it is not correct. If the last term of the trinomial has a minus sign in front of it, the two binomial factors cannot have <u>like signs</u> between their terms.

You remember why, don't you?

Return to page 103/1 and try the question again.

This is the correct answer.

Please make a note of the fact that a trinomial which has a minus sign before its middle term and a plus sign before the last term, factors into two binomials both of which have minus signs between their terms.

Please go on to question 10 below.

Question 10

Apply the proper principle and select the letter next to which a prime trinomial is written.

(A) $x^2 + 10x + 8$

(B) $y^2 + 7y + 10$

(C) $p^2 + 11p + 28$

(D) $x^2 + 6x + 9$

This is a pretty difficult problem. The reason that we included it is that we think that you can handle it. You should feel complimented when we ask you to do something difficult; it shows that we think highly of you.

Here is a small hint. Consider

$$(x - 2y)$$

as a single term. In fact, let

$$z = x - 2y$$

then the trinomial will become

$$z^2 + 2z - 15$$

Please go on from here.

Return to page 99/1 and try this question again.

We don't agree. Answer this question, and you will see why. When you multiply two binomials each having a negative sign between their terms, what will be the sign of the final term of the trinomial? Do you see your error?

Please return to page 110/2 and try this question again.

86
1

The only time when both signs between the terms of the binomials are positive is when <u>all</u> the signs between the terms of the trinomial are positive.

Please return to page 81/2 and try the question again.

86
2

The method of factoring

$$x^4 - 8x^2 + 12$$

is the same as that used for

$$x^2 - 8x + 12$$

However, the lead variable is

$$x^2$$

not x

Please return to page 102/2 and choose another letter.

Your choice leads to the trinomial,

$$x^2 + 19x + 18$$

But this trinomial can be factored. You don't think so?

Try $(x + 18)(x + 1)$

Please return to page 77/2 and reconsider the question.

We don't agree. You just can't pick any two factors of 10 and expect to have the correct answer. You must also check the sum of the inner and outer products.

Certainly $(-10r) + 1(r)$

does not equal $+3r$.

Please return to page 95/2 and try this question again.

88
1

Very good. You made the correct choice. If we enumerate all the pairs of positive integers whose product is 8 , we get

(8, 1) , (4, 2)

Since neither pair adds up to 10 , the trinomial cannot be factored.

Please proceed to question 11 below.

88
1 Question 11

A trinomial in the form

$$x^2 + 7x + k$$

might be factorable or it might be prime, depending upon the value of k .

Apply the proper principle and select the two values of k for which the trinomial

$$x^2 + 7x + k$$

can be factored.

(A) $k = 6$ or $k = 10$

(B) $k = 7$ or $k = 1$

(C) $k = 9$ or $k = 2$

(D) $k = 6$ or $k = 7$

89
1

It's true that the factors of 63 to be used are 9 and 7 , and it's also true that they should have different signs to product a negative 63 when they are multiplied.

However, when the outer and inner products are added, they should produce a sum of negative 2 .

Didn't you check that?

Please return to page 103/1 and reconsider the problem.

89
2

Almost right. However, as we told you before, almost right means wrong in mathematics. Concentrate your attention on the sum of the inner and outer products and you will discover your error.

Please return to page 98/1 and try this question again.

90/1

Maybe you are right. Let's check. Your choice results in the trinomial

$$x^2 + 9x + 18$$

which equals $(x + 6)(x + 3)$

Too bad, you are not right.

Please return to page 77/2 and try the question again.

90/2

We don't agree. Only the product of unlike signs results in a product with a minus sign in front of it. You realize that the constant (or last term) of a trinomial is equal to the product of the constants (or second terms) of the two binomials.

Please return to page 81/2 and try the question again.

What does

$$(r - 5)^2$$

equal? We have, in a previous lesson, learned how to square a binomial. Remember: we square the first term, then we take twice the product of the first and second terms and add the square of the second term. Thus,

$$(r - 5)^2 =$$
$$r^2 - 10r + 25$$

Please return to page 95/2 and try this question again.

Carelessness is the chief cause of mistakes in Algebra. Now listen carefully, and you will avoid making this mistake in the future. Observe, the last term of the trinomial is negative. The last term of a trinomial is obtained by multiplying the last terms of its binomial factors; hence, when the last term of a trinomial has a negative sign in front of it, the factors cannot have like signs between their terms.

Please return to page 98/1 and try this question again.

Congratulations! You succeeded in doing a pretty difficult problem.

Suppose we go over your solution of this problem. Consider

$$(x - 2y)$$

as a single term and let

$$z = x - 2y$$

Thus, our trinomial can be written as

$$z^2 + 2z - 15$$

This can be factored as

$$(z - 3) (z + 5)$$

Now, substituting $(x - 2y)$

for z

we get $(x - 2y - 3) (x - 2y + 5)$

This is the end of the segment. Before going on to the next segment, you should do problems 9 through 12 of the homework assignment.

Hand in your Punch Card.

Review your notes. Check to see if you have the following proto-types.

$$(x - a) (x - b) = x^2 - (a + b) x + ab$$

$$(x - a) (x + b) = x^2 - (a - b) x - ab \quad \text{if } a > b$$

$$(x - a) (x + b) = x^2 + (b - a) x - ab \quad \text{if } a < b$$

$$(x + a) (x + b) = x^2 + (a + b) x + ab$$

We don't agree. One of the letters does have the right answer next to it. Please reconsider each of the choices more carefully.

Return to page 103/1 and try this question again.

We don't agree. It is not enough that the product of the last two terms of the binomials equals the last term of the trinomial. You must also consider the coefficient of the middle term of the trinomial. Examine the sum of the inner and outer products more carefully.

Return to page 102/2 and try this question again.

94
1

This is the correct answer. Please go on to question 2 which follows.

Question 2

Apply the proper principle and select the letter which correctly completes the statement. The binomials which cannot be factors of

$$2x^2 + kx + 3$$

are:

(A) $(2x + 1) (x + 3)$

(B) $(2x + 3) (x + 1)$

(C) $(3x + 1) (x + 3)$

(D) $(x - 3) (2x - 1)$

94
2

You know that the coefficient of the linear term of a quadratic trinomial is equal to the sum of the inner and outer products of its binomial factors. Remember the F-O-I-L method? The letter a is equal to this sum. Please reconsider your choice.

Return to page 119/1 and try this question again.

This is the correct answer.

$$z^2 + 2z - 15$$

is equal to $(z + 5)(z - 3)$

Even if we didn't actually factor the trinomial, we can reason that the factors should be in the form

$$(z + a)(z - b)$$

because the sign of the product of the last terms,

$$-ab$$

would agree with the sign of the constant term of the trinomial,

$$(-15)$$

Please go on to question 13 below.

Question 13

Apply the proper principle and factor

$$r^2 + 3r - 10$$

Select the letter which labels the correct statement.

(A) $(r - 5)(r + 2)$

(B) $(r + 5)(r - 2)$

(C) $(r - 10)(r + 1)$

(D) $(r - 5)^2$

96
1

Very good. You made the correct choice.

We have

$$2x^2 - ax - b =$$

$$(2x + 1)(x - 5) =$$

$$2x^2 - 9x - 5$$

For the two trinomials to be identical, we compare them:

$$2x - ax - b$$

and see $-ax = -9x,$

whence $a = 9$

and $-b = -5$

whence $b = 5$

Please go on to question 6 below.

96
2

Question 6

Apply the proper principle and select the letter which correctly completes the statement:

If $3x^2 + kx + 6$

is written as the product of two binomials, the largest possible value of k is

(A) 11

(B) 19

(C) 21

(D) 9

Any time you make a choice in factoring a trinomial, this choice should be checked by multiplying the binomial factors. Their product must equal the original trinomial.

Did you really check your choice?

Please check it again.

Please return to page 102/2 and try this question again.

You must learn to profit by experience. Now you have been told many times that the product of two binomials with only plus signs between their terms will be a trinomial with only plus signs between its terms. Since the given trinomial has a minus sign before its last term, you know that your choice is incorrect.

Please return to page 110/2 and try this question again.

98/1

This is the correct answer.

Please go on to question 15 which follows.

Question 15

Apply the proper principle and factor

$$x^2 + 2xy - 48y^2$$

Select the letter which labels the correct statement.

(A) $(x + 8)(x - 6)$

(B) $(x - 8y)(x + 6y)$

(C) $(x - 8y)(x - 6y)$

(D) None of these.

98/2

We are lucky. Since the coefficient of the quadratic term and the coefficient of the constant term are each prime numbers, we do not have too many possibilities. You see

$$5x^2$$

can be written only as

$$(5x)(x)$$

Similarly, the number 3 can be written only as

$$(3)(1)$$

Keep in mind that by possible trial pairs of binomials, we mean only those pairs that will give us the correct coefficient of the quadratic term and the correct constant term.

5	3
1	1

We are sure that you can continue from here without our help.

Please return to page 113/2 and try this question again.

This is the correct answer. Please go on to the next question.

Question 17

Relate to the proper principle and factor

$$(x - 2y)^2 + 2 (x - 2y) - 15$$

Select the letter which labels the correct statement.

(A) This expression cannot be factored.

(B) $(x - 2y - 5)^2$

(C) $(x - 2y - 3) (x - 2y + 5)$

(D) $(x - 2y) (x - 2y - 2) - 15$

How do we determine the possible pairs of binomial factors? We consider the possible factors of the quadratic coefficient and the possible factors of the constant term. The questions that you must answer to yourself are:

What are the possible factors of 6 ?

What are the possible factors of 7 ?

You can finish the job from here without our help.

Please return to page 126/2 and try this question again.

100
1

VOLUME 9 SEGMENT 4 BEGINS HERE

Obtain a PUNCH CARD f[illegible]tor. In addition to the other identifying informari[illegible]urnished by you, you are asked to punch out the follow[illegible]

COLUMNS	48	and	50	4	4 (Sequence Number)
	54	and	56	0	4 (Type of Punch Card)
	60	and	62	0	9 (Volume Number)
	66	and	68	0	4 (Segment Number)

In your work in this Volume you have covered: factoring trinomial squares, multiplying two binomials at sight, and the reverse operation where a trinomial having the form

$$x^2 + bx + c$$

is factored into two binomials. Now you will go one step further and investigate the methods for factoring trinomials that have the form:

$$ax^2 + bx + c$$

That is, the co-efficient of the x^2 term is something other than one. This is the General Method for Factoring Trinomials.

READING: You should now turn to Page 259 in the Text and study the instructional material on that page and the following. Essentially, the method is one of trial and error; but you will see that there is a great deal of analysis involved which will reduce the number of trials. Judgment and common sense play an important part in this operation.

You will now be asked a series of questions to draw your attention to the more important points.

100
2

Question 1

Recognize which one of the following expressions is a general quadratic trinomial. Select the letter next to the correct answer.

(A) $x^2 - 6x + 19y$

(B) $3x^2 - x - 5x$

(C) $x^2 + y^2 + z^2$

(D) $8x^2 - 25x - 15$

It is important to be able to spot at a glance the binomials which cannot be factors of a given trinomial.

For example,

$$(3x + 1)$$

and

$$(x + 2)$$

cannot be the factors of

$$4x^2 + kx + 2$$

Why not? Because the product of the first terms of the binomials.

$$(3x)(x)$$

does not equal the leading term,

$$4x^2$$

of the trinomial.

Please return to page 94/1 and try this question again.

We don't agree. Why not multiply the binomials

$$(n + 2)$$

and

$$(3n + 1) \quad ?$$

Use the F-O-I-L method. Notice the sum of the inner and outer products. We are sure that after you do this, you will know the correct answer.

Please return to page 119/1 and try this question again.

102
1

Very good! You made the correct choice. You got an answer that was different from any of the three given choices and you showed confidence in the result you obtained by choosing "none of these." That's what we like to see, confidence. The factors of

$$x^2 + 2xy - 48y^2$$

are $(x + 8y)$

and $(x - 6y)$

a fact that can easily be verified by multiplying the two binomials.

Please go on to question 16 which follows

102
2

Question 16

Apply the proper principle and factor

$$x^4 - 8x^2 + 12$$

Select the letter which labels the correct statement

(A) $(x - 6)(x - 2)$

(B) $(x^2 - 6)(x^2 - 2)$

(C) $(x^2 - 4)(x^2 - 3)$

(D) $(x^3 - 6)(x - 2)$

This is the correct answer.

Please go on to the next question.

Question 14

Apply the proper principle and factor:

$$s^2 - 2s - 63$$

Select the letter which labels the correct statement.

(A) $(s - 9)(s - 7)$ (C) $(s - 9)(s + 7)$

(B) $(s + 9)(s - 7)$ (D) None of these.

Sorry, but you didn't make the right choice. Let's do a similar example.

Find the value of a and b if

$$3x^2 - ax - b =$$

$$(3x + 1)(x - 4)$$

Now we can find the product of the two binomials at sight by the

F-O-I-L METHOD

Thus, $(3x + 1)(x - 4) =$

$$3x^2 - 11x - 4$$

We compare this with

$$3x^2 - ax - b$$

The coefficient of the linear term in the trinomial with the letters a and b is -a . The coefficient of the linear term of the second trinomial is -11 . Hence,

$$a = 11$$

Need we go further? We have confidence that you can find the value of b without any trouble.

Please return to page 111/1 and try this question again.

104
1

You did not make the correct choice. Do you know why? The product of the constant terms of the binomials

$$(-3) \quad (-1)$$

does not equal the constant term of the trinomial,

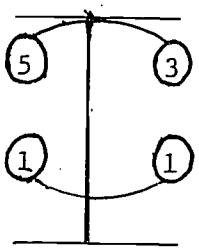
$$-3$$

Please return to page 110/2 and select another answer.

104
2

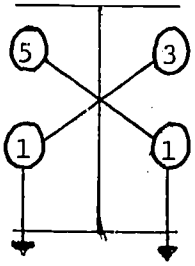
We don't agree. Note that both 5 and 3 are prime numbers. The only factors of 5 are , 5 and 1 . The only factors of 3 are , 3 and 1 . Thus, there are only a few possibilities.

15



1

15 + 1 = 16



3 5

3 + 5 = 8

Please return to page 113/2 and try this question again.

You made a hasty choice. One of the letters does have the correct answer next to it. We are not going to say any more. We believe that you would rather discover the correct answer by yourself.

Please return to page 119/1 and try this question again.

Did you notice that the last term of the trinomial is minus 1 ? You did ? Why did you choose binomial factors with plus signs between their terms ?

Please return to page 127/2 and select another letter.

IX

106
1

This question requires a little thought. What determines the values of k ? The value of k is the sum of the inner and outer products of its binomial factors. Now our job is to arrange the factors of

$$3x^2 + kx + 6$$

in such a way as to make the sum of the inner and outer products as large as possible. What are the factors of

$$3x^2 \quad ?$$

They are $3x$ and

x

What factors of 6 make one of the factors as large as possible? They are 6 and 1 . To obtain the largest value of k , we must arrange the binomial factors in a way that the largest factors; namely,

$3x$ and 6

will be multiplied together. Can you do this? Try.

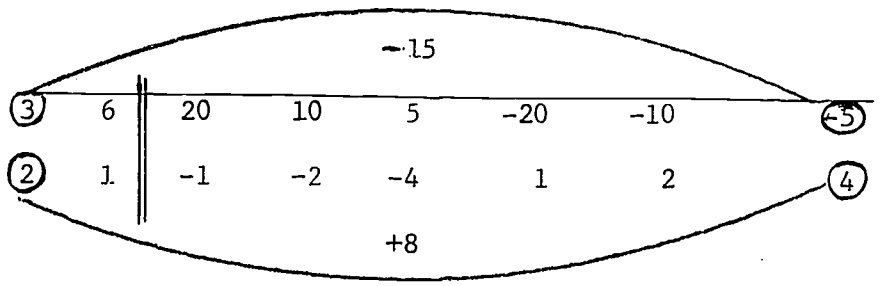
Please return to page 96/2 and do this question again.

106
2

Let us give you a hint on this problem. To factor:

$$6m^2 - 7m - 20$$

(1) We list the possible factors of 6 and -20 .



(2) We then try the possible combinations, adding the two products until we reach the combination that produces a total of -7 .

i. e. " (3) (-5) + (2) (4)

(3) Now use these numbers to write the binomials.

Please return to page 132/2 and finish the question

If you are right, then

$$(13m - 6) (m + 1)$$

should equal the given trinomial,

$$13m^2 - 7m - 6$$

Let us see, $(13m) (m) = 13m^2$

so that you are right, so far.

Also $(-6) (+1) = -6$

so that you are still right.

Now we must check the middle term of the trinomial. The sum of the inner and outer products is

$$(-6) (m) + (1) (13m) = + 7m$$

Too bad! The sum should be

$$-7m$$

Please return to page 120/2 and try this question again!

We don't agree. We think that you can do this problem, but the presence of the letters, a, b, k must have confused you a little. Let us help clear things up for you. We have,

$$12x^2 + 73x + k = (3x + 16) (ax + b)$$

We will work our way from left to right. The leading term of the trinomial is $12x^2$

One binomial starts with $3x$; the other binomial must, therefore, start with $4x$. You see it now; "a" must equal 4 . Now work with the sum of the inner and outer products, and you will be able to find the value of b . We are going to stop right there and let you have the fun of finishing this question by yourself.

Please return to page 118/2 and try this problem again.

Your choice is correct.

Let us review the situation:

To factor $6x^2 + 11x + 4$

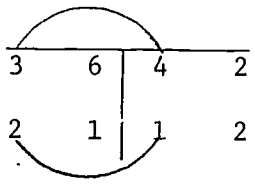
we write the factors of 6

and place them in the left half of the frame. (F)

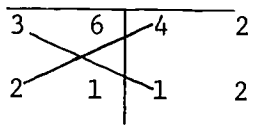
The factors of 4 are placed in the right half. (L)

(F)	3	6	4	2	(L)
	2	1	1	2	

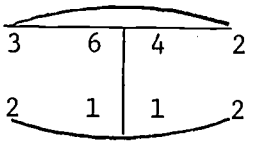
The different possible combinations can be found systematically as follows:



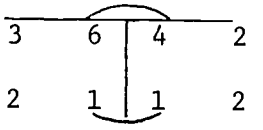
(3x + 1) (2x + 4)



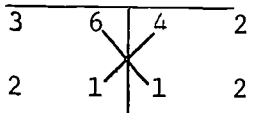
(3x + 4) (2x + 1)



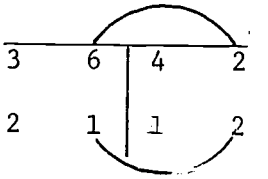
(3x + 2) (2x + 2)



(6x + 1) (1x + 4)



(6x + 4) (1x + 1)



(6x + 2) (1x + 2)

There are 6 possible combinations. Now proceed to question 10 on page $\frac{109}{1}$.

Question 10

Apply the proper principle and find the factors of

$$3z^2 + 7z + 2$$

Select the letter which labels the correct statement.

(A) $(3z + 2) (z + 1)$

(B) $(3z + 1) (z + 2)$

(C) $(3z + 3) (z - 2)$

(D) This is a prime trinomial and cannot be factored.

You made a mistake that you shouldn't have made. The last term of the trinomial has a plus sign in front of it. The two binomial factors that you chose have different signs between their terms. Need we say more ?

Please return to page 128/2 and try this question again.

110
1

Very good. The binomials

$$(3x + 1)$$

and

$$(x + 3)$$

cannot be factors of the trinomial

$$2x^2 + kx + 3$$

The product of the first terms of the binomial factors; namely,

$$(3x) (x)$$

must equal the quadratic term of the trinomial; which is

$$2x^2$$

Since this is not the case,

$$(3x + 1)$$

and

$$(x + 3)$$

cannot be the factors of the given trinomial.

Please go on to question 3 below.

110
2

Question 3

Apply the proper principle and select the letter which correctly completes the statement.

The binomials which can be factors of

$$8x^2 + kx - 3$$

are:

(A) $(4x - 3) (x - 1)$

(B) $(4x + 3) (2x + 1)$

(C) $(4x - 3) (2x + 1)$

(D) $(x - 3) (8x - 1)$

This is the correct answer. Please go on to question 5 which follows.

Question 5

Apply the proper principle and find the value of a and b if

$$2x^2 - ax - b = (2x + 1)(x - 5)$$

Select the letter which has the same answers that you got.

(A) a = -11, b = -5

(B) a = -9, b = -5

(C) a = 6, b = 4

(D) a = 9, b = 5

If you took the trouble to check your answer by multiplying the two binomials, you would have discovered that you made an incorrect choice. Don't you agree that it is more fun to check and be right than to be told that you are wrong?

Please return to page 127/2 and try this question again.

112
1

Very good. You made the correct choice. Let's go over the way we solved this problem. The " possible " factors of

$$3x^2 + kx + 6$$

are: $(3x + 3)(x + 2)$ $(3x + 1)(x + 6)$

$(3x + 2)(x + 3)$ $(3x + 6)(x + 1)$

Notice: We make sure that the " F " product is 3 and the " L " product is 6 . The outer and inner products vary and so does their sum.

If we take the sum of the inner and outer products of each of the above pairs of binomials, we find that

$$(3x + 1)(x + 6)$$

gives the largest value of k ; namely, 19 . What should be noted is that the binomial factors are so arranged that the largest possible factor of

$$3x^2$$

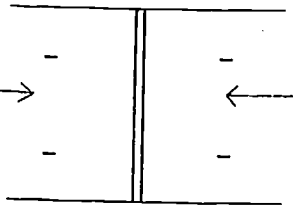
namely, 3x is multiplied by the largest possible

factor of 6 ; namely, 6 .

112
2

We will now explain a synthetic form to help you determine the possible factors of a trinomial.

Place the factors of x^2 coefficient here → " F "



← Place the various pairs of factors of the constant term here " L "

One of the " F " factors multiplies one of the " L " giving the " outer " product; the other member of the " F " pair multiplies the other member of the " L " pair giving the " inner " product. Different sums of the outer and inner product result. When the desired sum is found, the numbers are placed in the binomial form and the trinomial is factored.

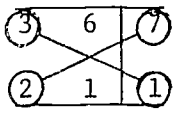
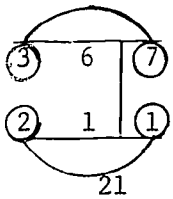
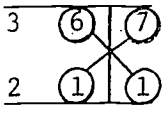
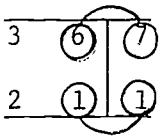
Please go on to page 113 .
1

For example:

The trinomial

$$6x^2 + kx + 7$$

has four possible sets of factors with all positive signs.

$6x^2 + 17x + 7$	$6x^2 + 23x + 7$	$6x^2 + 13x + 7$	$6x^2 + 43x + 7$
			
3	21	6	42
$(3x + 7)\ (2x + 1)$	$(3x + 1)\ (2x + 7)$	$(6x + 7)\ (x + 1)$	$(6x + 1)\ (x + 7)$
14	2	7	1
$k = 17$	$k = 23$	$k = 13$	$k = 43$

Please proceed to question 7 below.

Question 7

Apply the proper principle and find the number of possible trail pairs of binomial factors of

$$5x^2 + 8x + 3$$

Select the letter which labels the correct statement.

(A) 1 (C) 3

(B) 2 (D) 4

114
1

All you have to do to discover that you made an error is to find the sum of the inner and outer products of the choice you made. Does it equal

$$7z \; ?$$

It doesn't.

Please return to page 109/1 and try the question again.

114
2

Did you check the sum of the inner and outer products? You forgot. Let us check it together. The sum of the inner and outer products is

$$(3) \; (m) \; + \; (-2) \; (13m) \; = \; -23m$$

we need $-7m$

Please return to page 120/2 and try this question again.

Very good. You made the correct choice. We have to consider the possible factors of 6 and the possible factors of 7 . Now the possible factors of 6 are 3 , 2 and 6 , 1 . The possible factors of 7 are 7 , 1 . The total possible pairs of binomials are:

	or symbolically
(3x + 7)(2x + 1)	
(3x + 1)(2x + 7)	
(6x + 1)(x + 7)	
(6x + 7)(x + 1)	

3	6	7
2	1	1

7 can multiply any one of 4 numbers

7 x 6

7 x 1

7 x 3

7 x 2

Thus, 4 is the correct answer.

Please go on to question 9 below.

Question 9

Apply the proper principle and find the number of possible pairs of factors of

$$6x^2 + 11x + 4$$

Select the letter which has the same answer that you obtained.

(A) 8

(B) 4

(C) 6

(D) None of these.

116
1

Let's [illegible] ou are right. The sum of the inner and outer products of

$$(3x - 1)(2x - 3)$$

is

$$(-1)(2x) + (-3)(3x) =$$

$$-2x - 9x =$$

$$-11x$$

But in [illegible] n trinomial the linear term is equal to

$$-19x$$

Now you [illegible] ave done what we just did; that is, check yourself, and you would [illegible] found out that you are wrong.

Please [illegible] o page 128/2 and try this question again.

116
1

Did you [illegible] l the possible factors of a trinomial which has a quadratic term equ[illegible] to 3 and a constant term equal to 15 ? Well, you should have. Please do so now. You can use the synthetic analysis method.

Return to pa[illegible] 124/2 and try this question again.

You seem to have the right idea. Could you have been tripped up on this basic fact if

$$-5 = -b$$

then

$$b = 5 \ ?$$

Check your substitution and then return to page 111/1 and answer the question again.

The expression "greatest common factor", contains the word common. In this case common does not mean ordinary; it means something possessed by all. You are looking for the greatest factor of both terms of the binomial. For example, what is the greatest common factor of

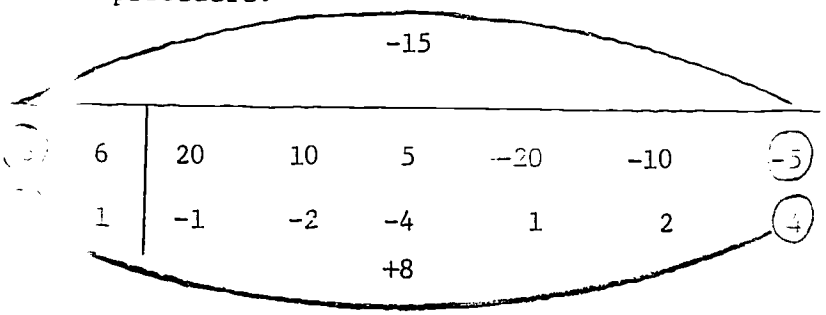
$$4ax^2 - 6ay^2 \ ?$$

The number 2 is a factor of both 4 and 6, and the letter a is a factor of both the first and second term. Hence, the greatest common factor is 2a.

Proceed again to page 141/2 and try the question again.

This [illegible] correct answer.

Let us [illegible] procedure.

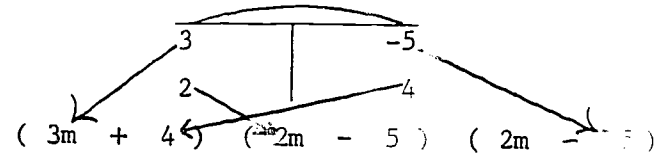


Since $(3)(5) - (2)(4) =$

$15 - 8 =$

7

which is the coefficient of the middle term; we have found the correct combination. They are now placed in the binomial form as follows:



This equals $6m^2 = 7m - 20$

Please go on to question 15 below.

Question 15

This problem is a brain twister. Can you meet the challenge? Apply the proper principle and select the letter which correctly completes the statement.

If $12x^2 + 73x + k =$

$(3x + 16)(as + b)$

then the values of a and b are :

(A) a = 3 , b = 4 , k = 16

(B) Cannot be determined

(C) a = [illegible] , b = 3 , k = 48

(D) a = [illegible] b = 3 , k = 48

This is the correct answer.

Please go on to question 4 below.

Question 4

Apply the proper principle and find the value of a if

$$3n^2 + an + 2 =$$

$$(n + 2)(3n + 1$$

Select the letter next to the correct answer.

(A) a = 5

(B) a = 7

(C) a = 6

(D) None of these.

We don't agree. What is the greatest common factor of

$$48x^2 - 3 ?$$

We are sure that you know that the answer is

3

After dividing each of the terms of the given binomial by

3

you are left with

$$16x^2 - 1$$

Don't you recognize this familiar expression?

Return to page 125/2 and try this question again.

120
1

Maybe you don't understand the question. You are asked to count the number of pairs of possible trial factors.

For example, one pair is

$$(5x + 3) (x + \quad)$$

Can you find any more

Return to page 113/2 and try this question again.

120
2

We now have a more difficult situation. Do you know why? Notice that both the coefficient of the quadratic term and the coefficient of the constant term have many factors. Thus, the possible factors of 6 are

3, 2 and 6, 1

The possible factors of 4 are

2, 2 and 4, 1

Please continue from here and list all the possible combinations of binomial pairs.

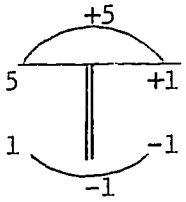
Return to page 115/2 and try this question again.

Let's review how we go about factoring a quadratic trinomial.

Examine it. One of the first things we should notice is the signs between its terms. In this question, the sign of the constant term is negative. What does that immediately tell us. The signs of the binomial factors cannot both have the same signs between their terms. Thus, the binomials must look like this:

$$(\ +\ )\ (\ -\ )$$

Use the synthetic algorithm observing the signs:



$+5 - 1 = 4$

Furthermore, before you select a letter in answer to this question, check your choice by multiplying the binomials. Their product must equal the given trinomial.

Please return to page 7
2 and try this question again.

Question 11

Apply the proper principle and find the factors of

$$13m^2 - 7m - 6$$

Select the letter which labels the correct statement.

(A) (13m - 6 + 1)

(B) (13m + 3 (m - 2)

(C) (13m - 3 m + 2)

(D) None of these

122
1

You were not listening when we told you that a trinomial with only plus signs between its terms cannot have binomial factors with minus signs between their terms. If we were mean, we would make you write the above fact 100 times, but we're not. So, write it down just once in your notebook.

Please return to page 109/1 and try this question again.

122
2

What is the greatest common monomial factor of

$$3x^3 + 18x^2 + 27x$$

You say, $3x$

We agree. Divide each of the terms of the trinomial by

$$3x$$

and examine the resulting expression carefully, especially the quadratic and constant terms. Do you recognize what kind of trinomial that is?

Return to page 150/1 and reconsider the problem.

We don't agree. One of the letters has the correct answer written next to it. Don't be lazy! Check each selection by multiplication of the binomials at sight, and you will find the correct answer.

Please return to page 122/2 and try this question again.

We don't agree. You correctly factored out the monomial factor 5 .

Thus,

$$5x^2 - 20 = 5(x^2 - 4)$$

The expression in the parentheses is the difference of two squares. We are not going to tell you how to factor this expression, but to ask you to look into your notebook and refresh your memory on how to factor the difference of two squares.

Please return to page 148/2 and try this question again.

Very good. You made the correct choice. We will enjoy doing this problem with you. We have

$$12x^2 + 73x + k =$$

$$(3x + 16) (ax + b)$$

Clearly, $(3x) (ax)$

$$12x^2$$

so that $a = 4$

Now let us put the value of a into the above indentity to get:

$$12x^2 + 73x + k =$$

$$(3x + 16) (4x + b)$$

The sum of the inner and outer products is:

$$(16) (4x) + b (3x) =$$

$$64x + 3bx =$$

$$73x$$

We see that $3b$ must equal 9, and $b = 3$.

Now, if

$$b = 3,$$

$$16b = 48 = k$$

Please proceed to question 16 below.

Question 16

Apply the proper principle and select the letter which correctly answers the question: What is the smallest coefficient of x which will make the prime trinomial

$$3x^2 + kx + 15$$

factorable?

(A) 18 (C) 46

(B) 14 (D) 8

This is the correct answer.

Factoring each term, we get

$$(3 \cdot a \cdot x \cdot x) - (3 \cdot 3 \cdot 3 \cdot a \cdot y \cdot y)$$

Comparison of the two terms shows that

$$3 \cdot a$$

appears in both; that is,

$$3a$$

is the greatest common factor.

Please go on to question 2 which follows.

Question 2

Apply the proper principle and select the letter which correctly completes the statement: After factoring out the greatest common factor from

$$48x^2 - 3$$

the resulting expression,

(A) Cannot be factored.

(B) Is the difference of two squares.

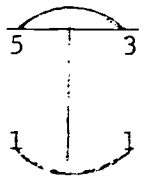
(C) Is a trinomial square.

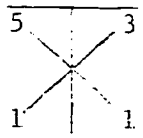
(D) Is a monomial.

126
1

This is the correct answer. $(5x + 1)(x + 5)$ $(5x + 3)(x + 1$

The possible trial pairs are: $(5x + 3)(x + 1)$ and $(5x + 1)(x + 3)$





We make sure that the First terms product stays as

$$5x^2$$

and the Last terms product is

$$3$$

By rearranging the binomials, keeping those products, we can change the sum of the Outer and Inner products.

Please proceed to question 8 below.

126
2

Question 8

Apply the proper principle and find the number of possible pairs of binomial factors of

$$6x^2 + 14x + 7$$

Select the letter which labels the correct statement.

(A) 4

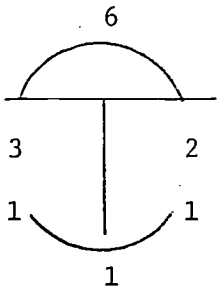
(B) 6

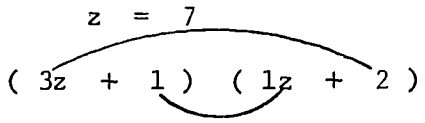
(C) 2

(D) 3

Let's use the symbolic analysis

$$3z^2 + 7z + 2$$





Therefore,

is the correct answer.

Please go on to question 11 below.

Question 11

Apply the proper principle and find the factors of

$$5x^2 + 4x - 1$$

Select the letter which labels the correct statement.

(A) (x - 1) (5x - 1)

(B) (x + 1) (5x + 1)

(C) (x - 1) (5x + 1)

(D) (x + 1) (5x - 1)

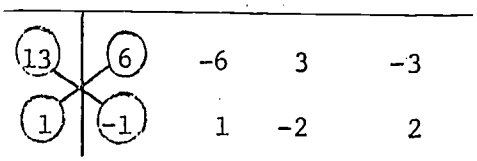
128
1

Very good. You made the correct choice. How did you arrive at your result? One way is to multiply each of the binomial pairs at sight. By doing this you find out that none of the choices equal the given trinomial. A still better way is not to look at the answers first, but to actually find the correct factors. Is that what you did? Good! You found that the correct factors of

$$13m^2 - 7m - 6$$

are $(13m + 6)$

and $(m - 1)$



$$-13 + 6 = -7$$

Please go on to question 13 below.

128
2

Question 13

Apply the proper principle and find the factors of

$$6x^2 - 19x + 3$$

Select the letter which labels the correct statement.

(A) $(6x + 1)(x - 3)$

(B) $(6x - 1)(x - 3)$

(C) $(3x - 1)(2x - 3)$

(D) None of these.

The word completely is written for a good reason. When you see this word it means that your job is not over until every part of your answer that can be factored is factored. Examine your choice and ask yourself: can

$$(x^2 - 1)$$

still be factored? Did you say yes? Well, then your job isn't over yet.

Please return to page 153/2 and try this question again.

Take another look at the choice you made. Can each of its terms be divided by the same number? if so, then your choice is incorrect.

Please return to page 151/1 and try question 7 again.

130
1

Very good! You made the correct choice. Clearly you noticed that the greatest common monomial factor of

$$3x^3 + 18x^2 + 27x$$

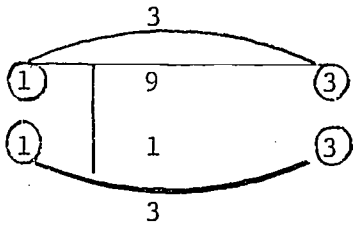
is $3x$

Thus, we have $3x^3 + 18x^2 + 27x =$

$$3x (x^2 + 6x + 9)$$

You can use the synthetic analysis to help you factor

$$x^2 + 6x + 9$$



$3 + 3 = 6$

Therefore, $x^2 + 6x + 9 =$

$$(x + 3) (x + 3)$$

or the expression in the parenthesis is a square trinomial. In other words,

$$x^2 + 6x + 9 =$$

$$(x + 3)^2$$

Please proceed to Question 4 below.

130
2

Question 4

Apply the proper principle and select the letter which correctly completes the statement: After removing the greatest common monomial factor from

$$6kx^2 - 21kx + 18k$$

the resulting expression,

(A) is a trinomial square .

(B) has one factor equal to (x + 2)

(C) has one factor equal to (2x + 3)

(D) none of these.

This trinomial can be factored. Try multiplying the other three pairs of binomials.

Please return to page 109/1 and try this question again.

Please check your choice by multiplying the two binomials. Their product must equal

$$13m^2 - 7m - 6$$

Please return to page 120/2 and try this question again.

132
1

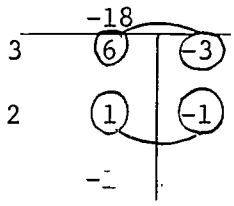
This is the correct answer.

Let's use the synthetic algorithm for this

$$6x^2 - 19x + 3$$

(1) Since the constant term has a plus sign, the middle signs of the binomials are the same.

(2) Since the sign of the $19x$ is minus, which must be arrived at by addition; the middle signs of both binomials should be minus.



$-18 \quad -1 \quad = \quad -19$

Therefore, the factors are

$$(6x - 1) \quad (x - 3)$$

Proceed to question 14 below.

132
2

Question 14

Apply the proper principle and find the factors of

$$6m^2 - 7m - 20$$

Select the letter which labels the correct statement.

(A) $(3m - 4) \quad (2m + 5)$

(B) $(3m - 10) \quad (2m + 2)$

(C) One of the factors must be $(3m + 4)$

(D) One of the factors must be $(2m + 5)$

How is this possible? The given expression is a binomial. You can't get a trinomial by removing a common factor from a binomial. Please read the question more carefully before making a decision.

Return to page 125/2 and try question 2 again.

We don't agree. In factoring an expression completely we first look for a common factor. After removing this factor, we examine the resulting expression to see whether it can be factored. I think that we better do a similar problem together.

Factor completely $2 - 32y^2$

Since each term can be divided evenly by 2, we can write,

$$2 - 32y^2 = 2 (1 - 16y^2).$$

Now, the expression in the parentheses is the difference of two squares. Hence, we have

$$2 (1 - 4y) (1 + 4y).$$

Please return to page 151/1 and try this question again.

This is the correct answer.

$32x^4 - 18x^2y^2$

$2x^2 (16x^2 - 9y^2)$ Remove common factor.

$2x(4x + 3y) (4x - 3y)$ Factor the difference of two squares.

Please go on to question 9 below.

Question 9

Apply the proper principle and factor the expression

$3x^2 + 12x + 12$

completely. Select the letter which labels the correct statement.

(A) $3(x + 4) (x + 4)$

(B) $3(x + 2)^2$

(C) $(3x + 4) (x + 3)$

(D) $(x + 2) (x + 2)$

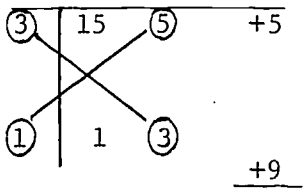
Very good. You made the correct choice. We are curious to know how you did this question. You could have listed all the possible factors of the trinomial disregarding the middle term. Thus:

(3x + 15) (x + 1)

(3x + 1) (x + 15)

(3x + 5) (x + 3)

(3x + 3) (x + 5)



Then you would discover that 14 is the smallest coefficient of the linear term. You did the question this way. Good!

Please go on to question 17 below.

Question 17

Apply the proper principle and find the factors of

$$12x^2 - 8xy - 15y^2$$

Select the letter which labels the correct statement.

(A) One of the factors is (2x - 3)

(B) The factors are (4x - 3y) (3x + 5y)

(C) One of the factors is (6x - 5y)

(D) None of these.

135
1

You lost something!

When an expression is written in factored form, all the factors must be written. Please find what you lost and add it to your answer.

Return to page 153 and reconsider the choices.
2

136
2

You correctly removed the common monomial factor

$$6m$$

but you made no attempt in factoring the trinomial in the parenthesis. It can be factored.

Please return to page 155 and try this question again.
2

You are not playing the game according to the rules. The rule is:

Remove the greatest common monomial factor first.

You don't need to be told that this factor is

$$2x^2$$

Please continue from here.

Please return to page 155
1 and try this question again.

We don't agree. You correctly observed that

$$(r + 2)$$

is a common factor.

However, when you divide each term of the given expression by

$$(r + 2)$$

you get $(4r^2 - 3r - 1)$

Please return to page 147
2 and try this question again.

138
1

You know what to do, but you were a little careless. After factoring out the common monomial factor 3 , you are left with the trinomial square

$$x^2 + 4x + 4$$

What are the factors of this trinomial?

Please return to page 134/2 and try this question again.

138
2

We don't agree. After removing the common monomial factor 2 , we have

$$2(3x^2 - 11x + 8)$$

The sum of the inner and outer products of the binomials you selected is

$-10x$

and you need $-11x$

Please return to page 156/2 and try this question again.

No, it can be done. If there is no remainder, the product of the divisor and the quotient equals the dividend. In other words, you are looking for a binomial which multiplied by

$$(x - 2)$$

will equal

$$2x^2 - x - 6$$

We'll say no more. Please continue.

Return to page 151/2 and try this question again.

You didn't factor completely. The trinomial

$$4r^2 - 3r - 1$$

can be factored.

Set up the synthetic analysis and be careful of the signs.

Please return to page 147/2 and try this question again.

Congratulations! You had the courage and confidence in your work to say " none of these. " Did you find the correct factors? You did. Examining the possible factors of 12 , we get

$$(12 , 1) , (6 , 2) , (4 , 3)$$

Similarly, the possible factors of 15 are:

$$(3 , 5) , (15 , 1)$$

Since the coefficient of the middle term of the given trinomial is not a very large number, common sense dictates that we avoid combinations using the factors

$$(12 , 1) \text{ and } (15 , 1)$$

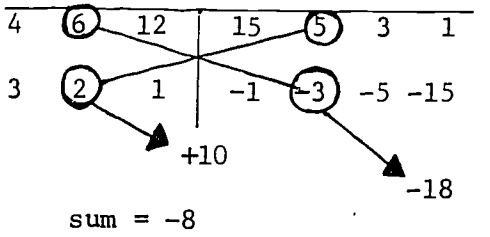
of 12 and 15 respectively. We first try (4 , 3) as factors of 12 and (3 , 5) as factors of 15 . This leads to incorrect results. We than try (6 , 2) as factors of 12 and (5 , 3) as factors of 15 .

Thus,

$(6x + 3y)(2x - 5y)$

$(6x + 5y)(2x - 3y)$

The synthetic analysis shows this:



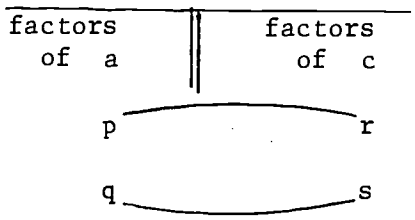
sum = -8

The second pair of binomials are the correct factors.

You have now finished Segment 4 . Hand in the PUNCH CARD.
You should have entered in your NOTEBOOK the following definitions and formulas.

general quadratic forms $ax^2 + bx + c$

synthetic analysis form for factoring trinomials



if $a = pq$

and $c = rs$

then $b = pr + qs$

that is $pqx^2 + (pr + qs) x + rs =$

$$(px + s)(qx + r)$$

You should be able to complete the following problems from your HOMEWORK ASSIGNMENT: Problems 13 through 16 .

VOLUME 9 SEGMENT 5 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	4 5	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 9	(Volume Number)
	66	and	68	0 5	(Segment Number)

Your READING ASSIGNMENT for this Segment is page : 261

SUPPLEMENTARY NOTES:

Up to this point, we studied problems that only required one procedure. You either had to remove a common monomial factor, or factor the difference of two squares; or factor a trinomial by trial. Now we are going to encounter expressions which will require more than one type of factoring to complete the job. Thus, a single expression may require removing a common monomial factor and the factoring of the difference of two squares. The important question is what do we do first? The first thing to do is to remove the common monomial factor if there is one.

You will now be asked a series of questions to draw your attention to the more important points

---

Question 1

Apply the proper principle and find the greatest common factor of

$$3ax^2 - 27ay^2$$

Select the letter next to the correct answer.

(A) 3x

(B) 9y

(C) 3a

(D) 3xy

142
1

We thought that we would never see this mistake again. When you divide

$$-x \text{ by } x$$

what do you get? Don't you remember that a number divided by itself equals one and not zero? Thus,

$$\frac{-x}{x} = -1$$

given that

$$x \neq 0$$

Return to page 153/2 and try this question again.

142
2

Did you check the choice you made by multiplying the two binomials? Well you should have. Do it now, and you will find out for yourself that you made a wrong choice.

Please return to page 155/1 and try this question again.

No. Remember the rule: If there is a common monomial factor, remove that factor first; then proceed to factor the remaining expression. Besides, did you really check your answer by multiplication?

Please return to page 148/2 and try this question again.

Almost right. Your mistake is in the monomial factor. Can you find it? We are sure that you can.

Please return to page 155/2 and try this question again.

144
1

If you had made this mistake a few weeks ago, we wouldn't have been too unhappy, but by now you should know better. Let's consider a similar problem. Remove the greatest common factor from

$$16x^2 - 4$$

Clearly 4

is the greatest common factor.

When you divide $16x^2$ by 4

you get $4x^2$

and when you divide -4 by 4

you get -1 not zero

Make sure that you will never make this mistake again!

Return to page 125/2 and try question 2 again.

144
2

What is the greatest common monomial factor of

$$6kx^2 - 21kx + 18k \quad ?$$

Can we factor out the number 3 ?

Can we factor out the letter k ?

Yes, you say. Let's do it and see what happens. We get

$$3k (2x^2 - 7x + 6)$$

All that is left to be done is to factor the trinomial in the parentheses. Please continue.

Return to page 130/2 and try this question again.

Sorry, you made a hasty choice. One of the letters does have the correct answer next to it.

Please return to page 151/1 and choose another letter.

You didn't remove the common monomial factor first. In the future, please examine the expression you have to factor, and if you see a common factor; remove that factor before you do anything else.

Please return to page 134/2 and try this question again.

146
1

If your choice is correct, then

$(x - 2)(2x - 1)$

equals $2x^2 - x - 6$

Does it?

Please return to page 151/2 and try this question again.

146
2

We don't agree.

If you multiplied the two factors of your choice, you would immediately discover that this product does not equal the original expression.

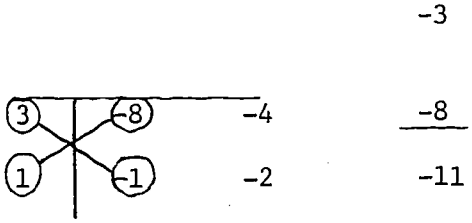
Please return to page 152/2 and try this question again.

Very good! You made the correct choice. After removing the common monomial factor 2 , we get

$$2 (3x^2 - 11x + 8)$$

We have to factor the trinomial in the parentheses. Did you have a little trouble with finding the correct factors?

Sure, but you have to keep trying.



The correct factors are

$$(x - 1)$$

and $(3x - 8)$

Proceed to question 13 below.

Question 13

Apply the proper principle and factor the expression,

$$4r^2 (r + 2) - 3r (r + 2) - (r + 2)$$

completely.

Select the letter which labels the correct statement.

(A) $(r + 2) (4r^2 - 3r)$

(B) $(r + 2) (4r^2 - 3r - 1)$

(C) $(2r - 1)$ is one of the factors.

(D) $(r - 1)$ is one of the factors.

148
1

This is the correct answer.

First you removed the common factor x .

Therefore, $x^3 - x$

became $x (x^2 - 1)$

Of course, you recognized

$(x^2 - 1)$

as your old friend the difference of two squares.

When it is factored it becomes

$$(x + 1) (x - 1)$$

So the complete answer is

$$x (x + 1) (x - 1)$$

Please go on to question 6 below.

148
2

Question 6

Apply the proper principle and factor completely

$$5x^2 - 20$$

Select the letter which labels the correct statement.

(A) $5 (x - 4) (x + 4)$

(B) $5 (x - 2) (x + 2)$

(C) $(x - 2) (x + 2)$

(D) $(5x - 4) (x + 5)$

149
1

You lost something on the way. In your final answer, every factor must be written. You omitted the common monomial factor.

Please return to page 134/2 and try this question again.

149
2

You did not complete the work. The trinomial in the parentheses can be factored.

Please return to page 156/2 and try this question again.

IX

150
1

This is the correct answer.

Please go on to question 5 which follows.

Question 5

Apply the proper principle and select the letter which correctly completes the statement:

After removing the greatest common monomial factor from

$$3x^3 + 18x^2 + 27x$$

the resulting expression,

(A) is a trinomial that can be factored by trial only.

(B) is a prime trinomial.

(C) is a trinomial square.

(D) None of these.

150
2

Yes, but you didn't finish the job. The expression in the parentheses is the difference of two squares and it, too, can be factored.

Please return to page 155/1 and try this question again.

This is the correct answer.

Please go on to question 7 which follows.

Question 7

Apply the proper principle and select the letter which correctly completes the statement:

If $3 - 27x$

is factored completely,

(A) one of the factors is (? 9x)

(B) one of the factors is

(C) one of the factors is (1 + 9x

(D) none of these.

This is the correct answer.

Please go on to question 11 which follows.

Question 11

Apply the reducing of fractions principle to find the quotient

$$\frac{2x^2 - x - 6}{x - 2}$$

without doing the actual division as a "long division" problem.

(A) It can't be done.

(B) $2x - 3$

(C) $2x + 3$

(D) $2x - 3$ and remainder 12 .

Yes. You made the correct choice. We are always happy when you get the correct answer to a pretty difficult problem. We are sure that you proceeded as follows:

You first factored out the common factor

$$(r + 2)$$

obtaining $$(r + 2) (4r^2 - 3r - 1)$$

Then you factored the trinomial; thus,

$$(r + 2) (r - 1) (4r + 1)$$

Please go on to question 14 below.

Question 14

Apply the proper principle and factor the expression

$$y^3 - y^2 + y - 1$$

completely. Select the letter which labels the correct statement.

(A) $y^2 (y^2 - 1)$

(B) $(y - 1)^2 (y + 1)$

(C) $(y - 1) (y^2 - y + 1)$

(D) $y^2 (y - 1)$

Very good! You didn't forget what you learned previously. The procedure that we follow is to first remove the greatest common monomial factor. Thus,

$$6kx^2 - 21kx + 18k =$$

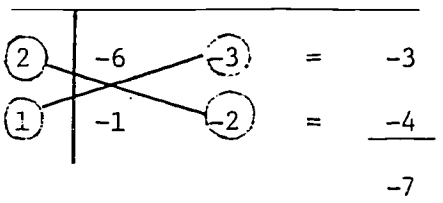
$$3k (2x^2 - 7x + 6$$

Then we proceed to find the factors of the trinomial in the parentheses using synthetic analysis:

$$2x^2 - 7x + 6$$

or

$$(2x - 3) (x - 2)$$



Hence, none of these is the right answer.

Please go on to question 5 below.

Question 5

Apply the proper principle and factor completely the expression

$$x^3 - x$$

Select the letter which labels the correct statement.

(A) $x (x^2 - 1)$

(B) $(x - 1) (x + 1)$

(C) $x (x - 1) (x + 1)$

(D) $x (x^2)$

154
1

You are not playing fair. You divided when the question asked you not to divide. To make things worse, you divided incorrectly.

Please return to page 151/2 and try this question again.

154
2

After removing the common monomial factor 2, we have

$$2 (3x^2 - 11x + 8)$$

The sum of the inner and outer products of the binomials you selected is

$$-14x$$

and you need

$$-11x$$

Please return to page 156/2 and try this question again.

This is the correct answer.

Let's review the procedure:

$3 - 27x^2 =$

$3 (1 - 9x^2)$ (removing common factor)

$= 3 (1 + 3x) (1 - 3x)$ (factoring difference of 2 squares)

Please go on to question 8 below.

Question 8

Apply the proper principle and factor completely the expression,

$$32x^4 - 18x^2y^2$$

Select the letter which labels the correct statement.

(A) $(8x^2 - 3y) (4x^2 - 6y^2)$

(B) $(x^2 - 3y) (x^2 + 6y)$

(C) $2x^2 (16x^2 - 9y^2)$

(D) $2x^2 (4x - 3y) (4x + 3y)$

This is the correct answer.

Please go on to question 10 which follows.

Question 10

Apply the proper principle and factor the expression

$$12m^3 + 6m^2 - 6m$$

completely. Select the letter which labels the correct statement.

(A) $6m (2m^2 + m - 1)$

(B) $6m (2m - 1) (m + 1)$

(C) $6 (2m - 1) (m + 1)$

(D) $6m (2m + 1) (m - 1)$

156
1

Very good! You made the correct choice. The job consists of finding a binomial which multiplied by

$$(x - 2)$$

equals $2x^2 - x - 6$

In other words, this was really not a problem in division, but a problem in factoring. You have to factor

$$2x^2 - x - 6$$

knowing that one of the factors is

$$(x - 2)$$

The other factor is

$$(2x + 3)$$

The reason for choosing this problem was to illustrate one of the uses of factoring. Some problems in division can be done more rapidly by factoring.

Please go on to question 12 below.

156
2

Question 12

Apply the proper principle and find all the factors of the expression,

$$6x^2 - 22x + 16$$

Select the letter which labels the correct statement.

(A) $2 (3x - 4) (x - 2)$

(B) $2 (3x^2 - 11x + 8)$

(C) $2 (3x - 2) (x - 4)$

(D) None of these.

Sorry, but

$$(2r - 1)$$

is not a factor of

$$(4r^2 - 3r - 1)$$

Do you know why? Suppose that

$$(2r - 1)$$

were a factor, what would the second factor have to be?

Please return to page 147/2 and try this question again.

Please check the sum of the inner and outer products of the two binomial factors. The sum should be "m" .

Return to page 155/2 and reconsider the question.

158
1

We agree with you that this is not an easy question. There are many possibilities, and if you are not lucky; you may not find the right one until the very end. Is it possible to reach the correct combination of factors sooner? We are going to say yes. How? Experience will do it. After you have done many trial factoring problems, you will reach the right combination of factors sooner. Common sense and good judgment also help. Now let's get down to the job before us. We have

$$12x^2 - 8xy - 15xy^2$$

Examining the signs between the terms of the trinomial, we immediately know that the binomial factors must be of the form (+)(-)
We now make a guess and try (4 , 3) and (5 , 3) for the factors of 12 and 15 respectively. We get.

$$(4x + 5y)(3x - 3y)$$

This combination is no good. We need a negative coefficient for the linear term. Interchanging the factors 5 and 3 , we get

$$(4x + 3y)(3x - 5y)$$

We must try another pair of factors of 12 . Let's try 6 and 2 . This may do it. Please continue from here. We are sure that you will find the correct factors.

Return to page 135/2 and try this question again.

158
2

You left something out. You see, in this case, you have the chance of checking your answer and finding out whether you left something out before it is too late. If you took the trouble to multiply (x - 2) and (x + 2) you would have seen that it does not equal $5x^2 - 20$

Please return to page 148/2 and try this question again.

A general quadratic trinomial is of the form

$$ax^2 + bx + c$$

where a , b , c are integers. For example:

$$3x^2 - 7x + 8$$

is such a trinomial. In this example,

$$a = 3$$

$$b = -7$$

$$c = 8$$

Please return to page 100/2 and try this question again.

We don't agree. One of the letters does have the correct answer next to it. Take a deep breath and start all over again!

Please return to page 115/2 and choose another letter.

160
1

We don't agree. Did you forget how to multiply two binomials at sight? I really don't think so. However, let's go over the rule. Consider the product,

$$(3y + 8)(2y - 5)$$

We multiply the first two terms,

$$(3y)(2y) = 6y^2$$

Then we take the sum of the inner and outer products,

$$8 (2y) - 5 (3y) = y$$

Finally we multiply the last two terms,

$$(8)(-5) = -40$$

Thus, $(3y + 8)(2y - 5) =$

$$6y^2 + y - 40$$

Please return to page 79/2 and try this question again.

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2

Congratulations! You made the correct choice. A problem of this type requires good thinking! I wonder whether you proceeded with this problem the same way that we did. Examining the given expression, we note that the first two terms have a common factor

$$y^2$$

Removing this common factor from the first two terms, we get

$$y^2 (y - 1)$$

We immediately notice that the terms in the parenthesis are the same as the remaining terms of the given expression. Thus, we have

$$y^2 (y - 1) + (y - 1)$$

Removing $(y - 1)$

as a common factor, we get $(y - 1)(y^2 - 1)$

But, the second binomial is the difference of two squares and can be factored. Hence, the final answer is

$$(y - 1)(y - 1)(y + 1)$$

or $$(y - 1)^2 (y + 1)$$

Continue on Page 161

You have now finished Segment 5 . Hand in the PUNCH CARD.

We suggest that you study all of your notes.

Important points:

Factoring trinomial squares

Using the F-O-I-L method to multiply two binomials at sight

Using the synthetic analyses to factor any trinomial which is not prime.

Good luck on your test!

You should be able to complete the following problems from your

HOMEWORK ASSIGNMENT: 17 through 20

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 9

Attach to Back Cover

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected. Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE	MISPRINT	CORRECTION	CHECK WHEN CORRECTION MADE
$\frac{118}{1}$	-15 ÷ 8 = 7	-15 ÷ 8 = -7	
$\frac{118}{1}$	(3m ÷ 4) (2m-5) (2m-5)	Delete one (2m - 5)	
$\frac{118}{1}$	$6m^2 = 7m - 20$	$6m^2 - 7m - 20$	
$\frac{150}{1}$	go to question 5	go to question 3	
	Question 5	Question 3	
$\frac{152}{1}$ & $\frac{152}{2}$	in right hand corner	in right hand corner	
$\frac{112}{1}$	$\frac{112}{2}$	should be one complete page	
$\frac{107}{1}$			
$\frac{114}{2}$	Please return to page $\frac{120}{2}$	Please return to $\frac{121}{2}$	
$\frac{131}{2}$			
$\frac{123}{1}$	Return to page $\frac{122}{2}$	Return to page $\frac{128}{2}$	
$\frac{121}{1}$	Please return etc.	Proceed to question 12 below	
$\frac{44}{2}$	= 21 p = 30 p	- 21 p - 30 p	